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INVESTIGATIONS ON PLANE REFLECTION GRATINGS WITH REFERENCE TO THEIR USE IN THE ABSOLUTE DETERMINATION OF WAVE-LENGTHS[1]

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I. INTRODUCTION AND DESCRIPTION OF THE GRATINGS

In 1894 A. A. Michelson succeeded in determining the wave-length of the red cadmium line at λ 6438 with his interferometer so accurately that the results of the separate experiments differed at most by 0.0066 tenth-meters, or 1:1,000,000 of the value.[2] Relatively to these absolute measurements, MM. Ch. Fabry and A. Perot,[3] by the same method and probably with the same accuracy, determined among others the wave-lengths of the mercury lines. They obtained the following values, reduced to 15° C. and 760 mm, which are important in connection wth the experiments to be described in this paper:

Hg Yellow lines	5790.659 5769.598
Green line	5460.7424
Indigo line	4358.343

[1] Translated from *Annalen der Physik*, **22**, 333, 1907. The paper was an abstract from the author's dissertation at Tübingen, 1906.

[2] "Détermination expérimentale de la valeur du mètre en longueurs d'ondes lumineuses," *Travaux et mémoires du Bureau international des poids et mesures*, **11**, 1895.

[3] "Détermination de nouveaux points de rep re dans le spectre," *Comptes Rendus*, **130**, 492, 1900.

They further determined the wave-lengths of D_1 to be 5895.932 for 15°, or 5895.960 for 20° C. The use of diffraction gratings for the absolute measurement of wave-lengths has been abandoned since these results appeared, aside from an investigation by Thalén,[1] which, according to the criticisms of H. Kayser[2] must be regarded as unsuccessful.

For purposes of comparison I give in the following summary the results for the sodium line D_1 obtained with gratings by different observers since 1886.

Observer	Maker of Grating	Material	Approximate Width of Grating in mm	No. of Rulings	D_1 for 20° C. and 760 mm
F. Kurlbaum*	Rutherfurd	Metal	43	29521	5895.86
(1886–87)	Rowland	Metal	42	23701	5895.98
L. Bell† (1885–87)	Rowland	Glass	30	12100	5896.18
	Rowland	Glass	30	8600	5896.23
	Rowland	Metal	100	29000	5896.15
	Rowland	Metal	100	40000	5896.17
R. Thalén (1898)	Rowland	Metal	51	29101	5895.976

* "Bestimmung der Wellenlänge Fraunhofersche Linien," *Wied. Ann.*, **33**, 159, 381, 1888.

† "On the Absolute Wave-Length of Light," *Am. Jour. Sci.* (3) **33**, 167, 1887; **35**, 265, 1888.

A study of the investigations of these observers shows that the differences between their results are considerably larger than they ought to be, as inferred from the errors of the instruments and the observations. According to the view of Kayser, the reason for this lies solely in the inadequate determination of the constants of the gratings: the width of the grating divided by the number of rulings, without taking into account the undoubted lack of uniformity of the rulings on account of technical defects. At the end of his discussion Kayser thinks the conclusion is justified that "it is impossible to determine the wave-length with gratings to an accuracy of 0.1 Ångström units."

The causes for the lack of uniformity in the distance of two successive rulings are doubtless due to errors of the dividing engine; to variations of temperature during the process of ruling; to change

[1] "Sur la détermination absolue des longueurs d'onde de quelques raies du spectre solaire," *Nova acta Upsal.*, 1899.

[2] *Handbuch der Spektroskopie*, **1**, 707.

or wear of the diamond, which surely can occur in ruling 29,000 or even 40,000 grooves, as in the case of the gratings used by Bell; and finally to the disturbance of a groove already ruled by the following one when the space is so small; for it seems inevitable that with from 300 to 700 grooves per millimeter the diamond will meet with less resistance on the side of the preceding groove than on the other side. The groove must to a certain degree injure the edge of the preceding groove, and itself also be injured. Finally, there might come into consideration a mechanical deformation of the substance of the grating in the course of time. (See papers by Bell and Thalén.)

If we share Kayser's view that all of the determinations above tabulated "were executed with about the same care, errors cannot be attributed to any of the observers, and they all refer to the same standards of length," then the question arises: If a grating is ruled under the most favorable conditions; in particular, if the sources of error just named are avoided as far as possible, will it then be possible to obtain a better agreement than heretofore between such gratings? And how will the values obtained compare with those of Michelson, and of Perot and Fabry? It must further be of importance and interest to discover as closely as possible the degree of error to be assigned to a grating. It will be necessary to show that the lack of uniformity of the space between the rulings may not exceed a definite limit according to theory, if it is not to render illusory the precision of the measures with the spectrometer and the comparator.

On the basis of these considerations I have, at the suggestion of Professor F. Paschen, carried out the following investigations.

I had for my experiments two reflection gratings made by Professor H. A. Rowland in accordance with Professor Paschen's wishes. The material of the two gratings is speculum metal, as employed in the other Rowland gratings. On a surface of 10.5 cm^2 a circle of about 9.5 cm was plane-polished by J. A. Brashear, of Allegheny, and on this plane surface, which was of the same size for the two gratings, the rulings were executed with Rowland's best machine. The two gratings were made in 1899 at the same time, and therefore are among the last ruled by Rowland. The ruled surface of the one which the experiment showed to be the better, which I shall designate hereafter as G_1, is 79 mm wide, and contains 3120 grooves, the height

of which is about 47 mm. The other grating, which will be designated as G_2, is 78 mm wide, and contains 3085 grooves of the same height as for G_1. Thus the two gratings have approximately the same constant. It is obvious from these facts that the errors above mentioned as probably the most serious are avoided to a greater degree than formerly. In spite of the small number of rulings, the width of my gratings is exceeded only by that of the two used by Bell. This gives the advantage that we can determine the constant of the grating—width divided by number of rulings—with an accuracy corresponding to that of the spectrometer measures; and, moreover, we increase the resolving power of the grating, as is shown by the relation

$$r=\frac{\lambda}{d\lambda}=\frac{b}{\lambda}\sin\delta,$$

where r is the resolving power, λ and $\lambda+d\lambda$ are the two wave-lengths which are to appear just separated, b the width of the grating, and δ the angle of diffraction. Further details and characteristics of the two gratings will be mentioned in connection with the separate experiments.

My experimental problem therefore included: (1) a precise investigation of the two gratings for their errors, in respect to the curvature of the surface and to the lack of uniformity of the space between the rulings; (2) a new absolute determination of the wave-length of some suitable spectral line by measurements with the comparator and spectrometer. (The latter measurements were actually made in advance of the investigation of the errors.) My experiments were carried out in the physical laboratory of the University of Tübingen in the period from December 1904 to Easter 1906.

II. INVESTIGATION OF THE CURVATURE OF THE SURFACE OF THE TWO GRATINGS

The most reliable method for this, which permits the test of the whole surface at once, is doubtless the use of interference fringes in the manner of the following experiments. The firm of Carl Zeiss, of Jena, placed at my disposal a quartz plate of cylindrical shape having a diameter of about 10 cm and a height of about 2.5 cm, one surface of which was guaranteed to be plane. The plane surface

of this quartz plate was laid upon three steel needles resting at equal intervals on the surface of the grating. The needles permitted me to vary the distance of the two surfaces and exclude any adhesion which might injure the surfaces. The objective of the telescope of the spectrometer was laid directly on the quartz plate, the whole apparatus being solidly built up on a horizontal surface. At the focal length of the objective a circular mirror of a few millimeters diameter was placed a trifle to one side of the optical axis of the objective, and this reflected the light of a sodium flame upon the objective, quartz plate, and grating. The interference phenomena in the sodium light reflected from the plane surface of the quartz plate and the grating could be seen directly beside the mirror. If the grating surface was accurately plane, it should be possible to make all the fringes disappear. For G_2 this was possible except for one fringe, which had a width of about 1 cm, and lay approximately parallel to the rulings; this was slightly curved above and below. The remainder of the field of view was wholly bright. The whole apparatus was left to itself for several hours for the purpose of eliminating any possible differences of temperature, but the results were the same. We may therefore conclude that the ruled surface of grating G_2 exhibits no differences of more than $\frac{1}{2}$ of a wave-length of sodium light (λ 5893, mean for D_1 and D_2).

Precisely the same experiment gave the following result for G_1: A position could be found at which a dark shimmer appeared solely close to the left and right edge of the ruled surface, while the remaining surface was uniformly bright. This justified the conclusion that no differences of more than $\frac{1}{4}$ of a wave-length occurred in the ruled surface of G_1.

Of course, this tells us a little as to the nature of the curvature; but we may assume it to be approximately cylindrical for G_2 with the axis parallel to the rulings, as may be perceived from the single interference fringe; we can only suspect the same to be true for G_1. We do not know, however, whether the grating is convex or concave, whence it is practically impossible to employ the theory of the curvature of diffraction gratings developed by Cornu.[1] But I will give

[1] "Sur la diffraction; propriétées focales des réseaux," *Comptes Rendus*, **80**, 645, 1875; "Etudes sur les réseaux diffringents;" "Anomalies focales," *ibid.*, **116**, 1215,

a geometric representation which permits us to judge of the amount of the error which might be caused, particularly for my two gratings, by this curvature. Let the curvature of the grating be considered cylindrical, and the section $A_1 A_2$, an arc, represent the ruled surface in Fig. 1. Now, if we assume the grating to be free from errors in regard to surface, and consider it first at the point A_1 and then at the point A_2, perpendicular to the radius of curvature of the concave grating, then the two directions will form the angle γ. If we now let parallel light fall on the grating, that is E_1A_1 be parallel to E_2A_2, the rays will make an angle of incidence $E_1A_1N_1=a_1$ at the first position of the grating; and the angle of incidence $E_2A_2N_2=a_2$ at the second position. We then have the following relations:

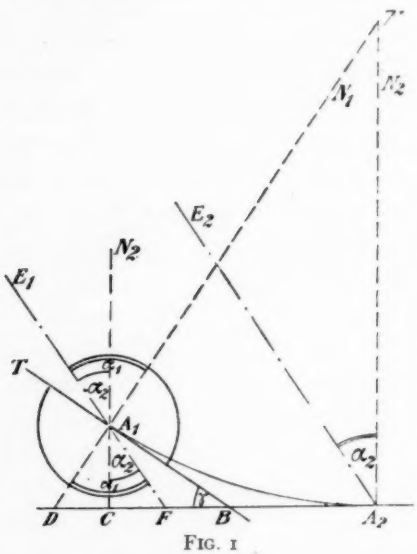


FIG. 1

$$E_1A_1N_2=FA_1C=a_2\,,$$
$$E_1A_1B=TA_1F=R+a_1=R+a_2+\gamma\,,$$
$$a_1=a_2+\gamma\,.$$

Applying this now to the actual conditions, the greatest differences of the surface of the grating will be: for G_1, $\frac{\lambda}{4}$; for G_2, $\frac{\lambda}{2}$; whence $A_1C=\frac{\lambda}{4}$ or $\frac{\lambda}{2}$.

If we then make A_1B equal to one-half the width of the grating we obtain for G_1,

$$\sin\gamma_1=\frac{2\lambda}{4b}=\frac{\lambda}{2b}\,,$$

1893; "Sur diverses méthodes relatives à l'observation des propriétées appelées 'anomalies focales' des réseaux diffringents," *ibid.*, 1421, 1893; "Vérifications numériques relatives aux propriétées focales des réseaux diffringents plans," *ibid.*, **117**, 1032, 1893.

and for G_2,

$$\sin \gamma_2 = \frac{2\lambda}{2b} = \frac{\lambda}{b} .$$

We thus compute γ for G_1 to be about $\frac{3}{4}''$; for G_2, about $1\frac{3}{4}''$. The angle of incidence therefore may differ in an extreme case for the whole surface of the grating by this amount, whence we may safely draw the conclusion that the two grating surfaces depart so slightly from the plane that they may be regarded as perfectly plane in practice; that is, within the accuracy of the measurements.

III. INVESTIGATION OF THE TWO GRATINGS FOR IRREGULARITIES OF THE SPACE BETWEEN THE RULINGS

The practical problem is to establish quantitatively the amount of the irregularity in the separation of the rulings, whether it be caused by the curvature of the surface or by the other defects mentioned previously. It is necessary for this purpose to examine a few rulings at a time successively over the whole ruled surface. Bell[1] evidently had in mind the method that I have employed, but, as his description indicates, he does not seem to have accomplished his purpose in consequence of technical difficulties. He therefore calibrated his gratings with a comparator, with the consciousness that the calibration only approximately disclosed the errors. I have succeeded, however, in obtaining with an angle of 26°, and using the method of auto-collimation, a diffraction image from as few as 40 rulings, which appeared sufficiently well measurable when photographically recorded. The 40 rulings were cut out by a slit parallel to the rulings, which was moved along in front of the surface of the grating. The errors of spacing then appeared to the eye as the oscillations of the spectral line with respect to the cross-hair in the eyepiece when this slit was moved with sufficient rapidity. Thus my method of detecting the errors of the grating shows directly their effect on the pattern, which is the essential matter, and permits me to determine, for each desired limited region of the grating, the differences of spacing. Thus the method should be preferable to that of Bell.

The spectrometer used was the same one with which the measures

[1] *Am. Jour. Sci.* (3) **35**, 357, 1888.

of the angle of diffraction were made. It was made by R. Fuess, of Berlin-Steglitz, according to Professor Paschen's designs, and for the particulars I would refer to my dissertation. The essential thing to be remarked is that the two Porro microscopes read directly on the graduated circle to 1″, and permit an estimate to $\frac{1}{10}$″. The original telescope and collimator could not be used on account of too small aperture, and so they were replaced by others made by Zeiss with aluminum tubes. The objectives were apochromatic triplets with an aperture of about 70 mm and focal length of 43 cm. The ordinary eyepiece of the telescope could be replaced by a slit-eyepiece which I used for the majority of my experiments. The lower half of this eyepiece is occupied by the slit and the total-reflection prism, which throws the light through the objective upon the grating; a reticle occupies the upper semi-circle.

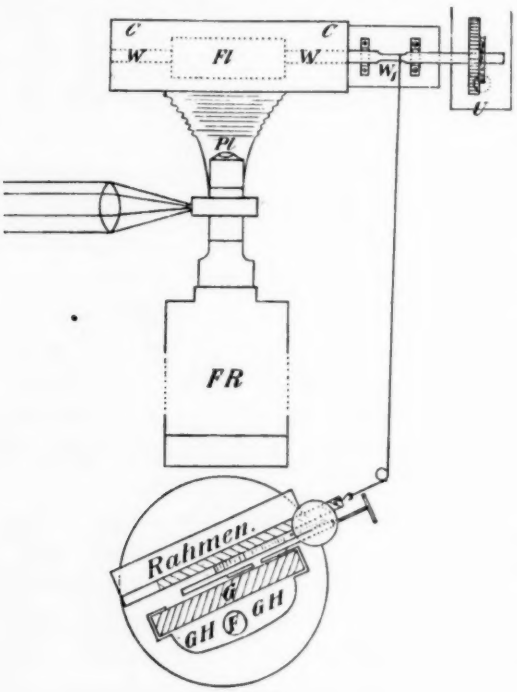


FIG. 2

A mercury arc *in vacuo* served as the source of light. Professor Paschen had several lamps constructed according to Arons' principle, which gave an excellent light with 60 volts and 4½ amperes, and could run for 20 minutes without interruption for cooling.

Fig. 2 shows the arrangement for cutting out a small number of rulings and for photographically recording the resulting oscillations of the spectral line employed. The adjustment of the photographic experiment is carried out in the following steps. The grating *G* is

placed in the holder *GH*, with the spectrometer table removed. A thermometer *T*, graduated in tenths of degrees, and permitting estimates within a few hundredths of a degree, was attached to this grating-holder in a mercury capsule. Care was taken that the variations of temperature did not exceed 0°.1 during an experiment, which often lasted through several hours. The grating was now adjusted for auto-collimation. The slit for cutting out the small number of rulings was adjusted parallel to the rulings. The graduated circle was turned until the line to be photographed appeared in the desired order beside the vertical mark in the eyepiece which replaced the cross-hair. The films employed were hardly sensitive to the lines λ 5460 and 5790, so that I was compelled to use the blue line λ 4358, which was photographically effective. The eyepiece was replaced by a Zeiss "planar" lens, 1:4.5, of focal length 35 mm. The camera *C* was displaced in the direction of the telescope's axis, so that subsequently the film was moved past in the focus of the planar lens. Everything was screened off in front of the film by black cardboard, except a horizontal strip 1.5 mm high, in which appeared the mark and the spectral line. The film, of English manufacture, 4 mm wide, was placed in the camera on the roll *W*, and then the experiment could be begun. In principle it consisted in moving the slit directly in front of the grating by an improvised clockwork *U* turning the shaft W_1 and simultaneously unrolling the sensitive film. At the beginning and end of each experiment the image of the uncovered grating is photographed; and at the conclusion it is repeated after a change of angle of 60″, so that the variations could afterward be measured according to their amount in seconds. In my dissertation I have treated *in extenso* of certain possible outside errors which might equally produce apparent variations of the spectral image on the film, not due to the irregularity of the grating-space. I have there described how I avoided such errors, principally by the greatest possible variations in the experiments, as they might produce very serious confusion.

At an angle of incidence of 26° 1′, the limiting measurable visibility was about 40 rulings; and for this the necessary exposure was so long that I had to content myself with taking short sections of equal distances successively over the grating, while with the wider

slit, for which a decidedly shorter exposure was sufficient, I could readily cover the grating in a few hours.

In the measurement of the films the point was to be able to measure the variations of distance between the spectral image and the mark corresponding to the displacements of the moving slit. For this purpose I calibrated the films perpendicularly to the image of the mark on a dividing engine. I had for the measurement an ordinary microscope with micrometric stage and rotating plate. The ratio of enlargement was reduced by a suitable combination of lenses to about 1:2.5. The micrometer screw had a pitch of 0.2 mm, and one division of the head was equal to 0.002 mm. By the use of the well-known method of adjustment the shortest distance of the spectral line and mark was always measured. Subsequently the measurements were arranged graphically, the displacements of the slit being platted as abscissae, and the corresponding distances between the mark and spectral line gave the ordinates, for which the variations were exhibited by a curve. The most important of my experiments were the following:

A. Six measurements by sections with a narrow slit, 1 mm being about equal to 40 rulings for grating G_1.

$\lambda = 4358$. 51st order, to the left, $i = 26° 1'$. To each section corresponded an exposure of 40 minutes and 360 revolutions of the smallest wheel of the clockwork, whence 3.5 mm of displacement of the slit was equal to about 141 rulings, so that the whole number of rulings that were affected was $141 + 40 = 181$, the displacement of the slit of about 9.3 mm corresponding to one revolution of the largest wheel of the clockwork. The slit was displaced about 10 mm between every two sections.

I will content myself with publishing the numerical data of section 4 for illustrating the measures. I repeated all the measurements as shown in column 5*a;* each figure of this column denotes the mean of three settings on the center of the spectral line: column 5*b* gives the mean of these two measures. The significance of the other columns is as follows: The first gives the temperature during the exposure; the second, the length of the strip of film used; the third, the displacement of the grooves for one interval of calibration; the fourth, the interval of calibration at which the measures were made;

column *6a* gives the largest difference of distance between the mark and the diffraction pattern within one section, which is converted into seconds of arc in column *b*, while *c* contains the corresponding difference of the constants of the grating.

FIG. 3

The accompanying Figure 3 is a copy of the section with the mark at the right and spectral line at the left; but unfortunately the copies do not satisfactorily represent the originals.

EVALUATION OF THE DISTANCE OF 60″

THE ENTIRE GRATING, JANUARY 17, 1906

POSITION	MARK RIGHT EDGE	SPECTRAL LINE			DISTANCE FROM MARK TO CENTER OF LINE	DIFFERENCE II−I
		Right Edge	Left Edge	Center		
I	2.081	7.936	8.858	8.397	6.316	0.817
II=I+60″	2.081	8.752	9.676	9.214	7.133	

The difference between the mark and the mean diffraction pattern amounts to 6.316 mm on the film. A displacement of the spectral image on the film of 0.817 mm corresponds to a rotation of the graduated circle of 60″.

MEASUREMENT OF A SECTION

1 *t*		2	3	4	5. DISTANCE OF MARK AT RIGHT EDGE FROM CENTER OF LINE IN MM		GREATEST DIFFERENCE		
a	*b*				*a*	*b*	*a*	*b*	*c*
		mm							
20°.93				1	6.544 492	6.518			
				2	6.578 572	6.575			
				3	6.478 432	6.455			
				4	6.290 228	6.259			
20.95				5	6.246 266	6.256			
				6	6.274 260	6.267			
	20°.95			7	6.338 340	6.339			
		15.1	8	8	6.274 260	6.267			
				9	6.280 290	6.285			
20.97				10	6.273 309	6.291			
				11	6.353 361	6.357			
				12	6.362 350	6.356			
				13	6.354 358	6.356			
				14	6.266 268	6.267			
				15	6.199 217	6.208			
20.93				16	6.218 232	6.225			
				17	6.217 218	6.217₅			

2 − 15 = 0.367 mm

28″

70.6 Å. U. = 1″.3600

The following six curves exhibit the variation of the ruling-space for all the six sections of the experiment. The fourth curve from the top in Fig. 4 corresponds to section IV, the measurements of which are given in full. The straight line corresponds in all cases to the diffraction pattern of the full surface of the grating. The accuracy

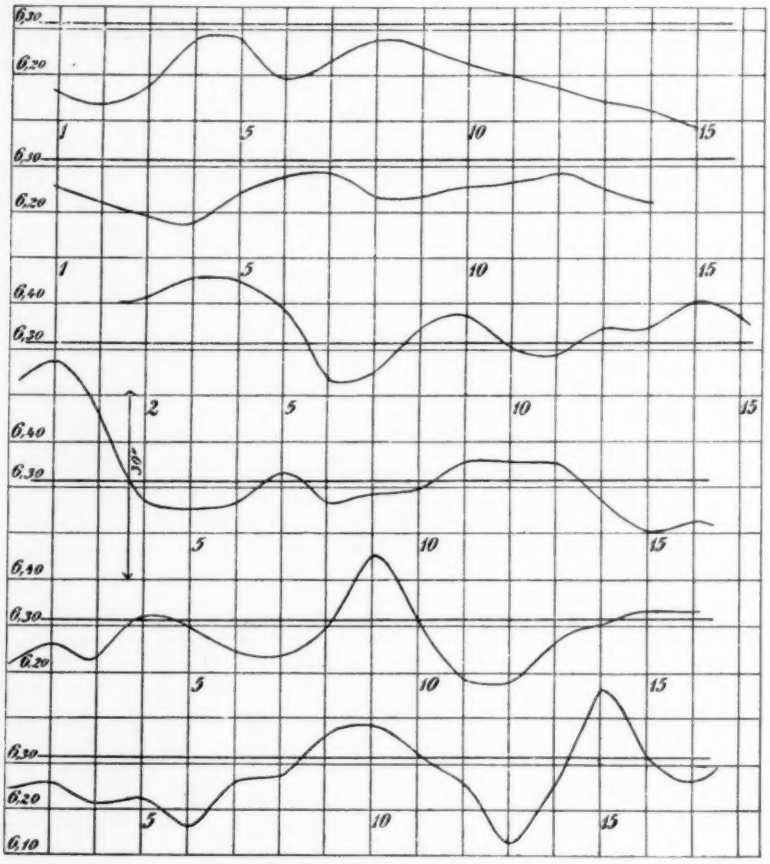


FIG. 4

of measurements is seen to be not very great on account of the indistinctness of the spectral line; the error can amount to a few hundredths of a millimeter, equal to about 4″, but in general would hardly exceed 2″. The previous experiment shows that by the isolation of a small number of rulings we have to do with very considerable variations;

that is to say, *we cannot properly speak of a constant of the grating.* The largest difference between mark and spectral line in the whole experiment is Fig. 4, fourth curve; 2 – first curve; $14 = 33''.5 = 84.5$ Å.U. (1 : 3000) difference of the space between rulings. It is a striking fact that sections I and II lie throughout below the line of the image of the whole grating, whence it would follow that at this edge of grating G_1 the constant was too great throughout. The later experiments with wider slit confirmed this fact. In fact, Bell found something similar in his calibrations.

According to this experiment periodic errors could not be proven to be present; if they actually exist, they are apparently so affected by other errors of the rulings, which operate first to intensify the phase (see fifth curve at 9) and then to weaken it, that they no longer have any validity as such, and have no practical signification. Rowland's theory for the periodic errors of the spacing of gratings therefore cannot be applied.[1]

An experiment *B* then followed, in order to determine in what ratio the oscillations decrease on increasing the number of rulings which are simultaneously effective. The section from the middle of the grating G_1 was used in precisely the same position as in experiment *A*. For a definite section of about 25 rulings—to select at random an example of the figures—the oscillation changed as follows:

No. of Rulings Effective	Oscillations
100	0.021 mm = 1".6 = 4.0 t.-m. difference from *c*
80	0.043 = 3.2 = 8.1
60	0.110 = 8.3 = 20.9
40	0.140 = 10.5 = 26.5

No relation can be proven to exist between these figures, but it could scarcely be present as long as the spacing does not vary according to some law.

I then made for grating G_2 two experiments similar to *A* and *B*. The result was quantitatively and qualitatively essentially the same as for G_1. I therewith gave up experimenting with the narrow

[1] "Gratings in Theory and Practice," *Astronomy and Astrophysics*, **12**, 129, 1893.

SUMMARY OF EXPERIMENTS WITH WIDE SLIT PASSING OVER ENTIRE GRATING

Grating	Date	Designation	Width of Grating-Slit	Range of Temp. During Exposure	Position of Grating	Diameter of Roll †	Portion of Grating Used	Length of Film	60″=mm Difference	Position of Spectral Image for Open Grating	Position of "Center" Line‡	Greatest Variation		
												In mm	In ″	Effect on ϵ
			mm			mm	mm	mm						
G_1	11, III 1906	C	5	0°.08	R_2 right 51st Order right $i=26^\circ\ 1'$	8	57.54	211.8	0.831	2.481	2.491	0.115	8″.3	20.9 Å. U. (1:12000)
G_1	11, II 1906	D	5	0°.09	R_1 right 51st Order right $i=26^\circ\ 1''$	8	53	188.3	0.81	4.350	4.350	0.137**	10″.2	25.8 Å. U. (1:9700)
	19, III 1906	E	5	0°.05	R_2 right 52d Order right $i=26^\circ\ 34'$	4	52	120.5	0.81	1.948	1.981	0.129	9″.6	24.2 Å. U. (1:10000)
G_1	15, III 1906	C_1*	5	0°.04	Position as for C	8	....	....	0.79	1.866	.....	.....	....	.....
G_1	20, III 1906	E_1*	2.5	0°.10	Position as for E	4	....	....	0.79	1.960	.....	.....	....	.....
G_2	1, III 1906	F	5	0°.14	R_2 right 39th Order right $i=19^\circ\ 34'$	8	55.1	194.6	0.86	2.934	2.956	0.145**	10″.1	35.0 Å. U. (1:7000)
G_2	23, III 1906	G	3	0°.07	R_1 right 39th Order left $i=19^\circ\ 35'$	4	48.6	135.6	0.76	3.224	3.250	0.116	9″.2	31.8 Å. U. (1:7800)

* Edges of grating; entire grating in other cases.
† Axis on which film is rolled.
‡ Mean of all values of distance of mark from line.
** Edge values omitted.

slit, hence with the small number of grooves. But it was of importance, and in practice of interest, whether on selecting a considerable number of rulings (I usually took 200), differences as compared with the spectral image of the entire grating would appear, and in what amount. This experiment, on account of the short exposure required, could be extended over the whole grating, which was of course important. I made such experiments for both gratings in the most varied positions, in order to be sure of the outside errors previously described as possible. The spectral line was so clearly and sharply defined that I could very readily set on its edges. The accuracy of the setting is therefore decidely greater than for *A* and *B*. These experiments are designated in the original by "C" to "G" with their results in the accompanying table. A peculiarity appeared for G_2, that at one edge the spectral line appeared to be split and superposed upon itself. "F" showed this so strongly as to affect the measures. The experiments give the result that for both gratings, even with a wide slit where as many as 200 grooves are affected, still we have to do with noticeable oscillations of the diffraction pattern. I shall therefore appropriately discuss this further in speaking of the results for the next experiment, the absolute measurement of the wave-lengths of the mercury lines λ 5460 and 5791.

The fundamental formula for the determination of a wave-length by the grating method is given by the theory of diffraction of light as

$$m\lambda = c\left(\sin(\delta+i)-\sin i\right).$$

Here λ denotes the wave-length to be determined, c the grating-constant, i the angle of incidence, δ the diffraction angle of the light employed, m the order.

IV. THE CONSTANT OF THE GRATING

In spite of the results of the previous section, I was at first obliged to avail myself of the customary practical definition of the constant as the width of the grating divided by the number of spacings. I had a small Zeiss comparator for measuring the width; with a magnifying power of about 60, two rulings (of which there were about 40 to one millimeter) were well separated, and permitted me to detect the details of their form. With suitable illumination the unruled portion

of the surface appeared dark, and upon it each ruling was projected in the form of the diamond which had ruled it as a system of four bright parallel lines; on closer examination these had a trough-like appearance. The grooves of G_1 appeared more brilliant and sharper than those of G_2. It was easy with this large grating-space to count the grooves, and I found

3119 spaces on 79.0 mm for G_1;
3084 spaces on 78.2 mm for G_2.

In the course of my experiments it appeared, however, that the small Zeiss comparator was not adapted to the large degree of precision required, whence at the conclusion of the experiments, both gratings were sent to the Normaleichungskommission at Berlin, where the widths were measured with the apparatus there available. The result of the four series of measures made, each of which included 32 experiments, is as follows:

Grating	Width Reduced to 17°	Deviation from Mean of All	Largest Range of Temperature in a Set of Measures
G_1.....	79 mm + 50.1 μ	0.5 μ	17°66 − 16°01
G_1.....	+ 50.7	0.3	17.14 − 15.97
G_2.....	78 mm + 162.2	0.4	17.96 − 16.35
G_2.....	+ 163.1	0.4	17.58 − 16.41

The commission remarked: "The uncertainty of the absolute amount of the widths of the gratings is to be taken at about $\pm\frac{1}{2}\,\mu$ for room temperature of +17° C. If observations are made at another temperature, there will be added to this uncertainty the possible error in the assumed coefficient of expansion, which we should estimate as about $\pm 0.5\,\mu$ for 1° and one meter's length."

The final result for the width of the gratings, that is, for the distance of the center of the first to the center of the last ruling, measured perpendicularly to the rulings, is:

$G_1 = 79.0504$ mm at 17° C.,
$G_2 = 78.1629$ mm at 17° C.

Therefore the grating-constant is:

for G_1, $c = 25.3447_9\,\mu$ at 17°,
for G_2, $c = 25.3446_5\,\mu$ at 17°.

The uncertainty of these two absolute values would therefore probably not be greater than $\pm 0.0002\ \mu$ for 17°.

The coefficient of expansion ϵ of the substance of the grating I determined by comparison of the widths at different high temperatures; and again by the method given by F. Kurlbaum,[1] where I adopted $n = 1.0002763$ from the determinations of Kayser and Runge[2] as the index of refraction of the not dry air at 16°, and 760 mm for the wave-length of the light I used, *Hg* λ 5460. The coefficient of expansion for the two gratings resulted in the mean $\epsilon = 18.26\ \mu$ for 1° C. and one meter, with a mean error of $\pm 0.10\ \mu$.

The spectrometer measurements which I made use of were conducted within the range of temperature 15° to 19° for G_1, and 13° to 23° for G_2 (once at 11°.5); of these the measurements of the width of the grating were made between 15°.7 and 17°.60 for G_1 and between 16°.35 and 17°.96 for G_2. Accordingly the greatest possible difference of temperature is in round numbers 6° between the measurements of the width of the grating and those with the spectrometer.

But an error of $0.5\ \mu$ in the coefficient of expansion with such a maximum difference of 6°, would involve an error of the grating of only 3 parts in a million. This error is therefore smaller than the uncertainty arising from the errors of observation, whence we may regard the coefficient of expansion as determined with sufficient accuracy.

V. MEASUREMENT OF THE DIFFRACTION ANGLE

The spectrometer, grating-holder, and thermometer which I used have already been described in Section III. The equation for determining the wave-lengths from grating measurements, derived from Huygen's principle, is:

$$m\lambda = c[\sin(\delta + i) - \sin i] \qquad (1)$$

If the surface of the grating is perpendicular to the incident ray, $i = 0$ and (1) becomes

$$m\lambda = c \sin \delta\ . \qquad (2)$$

If we observe on the other side of the direct image the order corresponding to that of equation (1), we may write it

$$m\lambda = c[\sin i + \sin(\delta - i)]\ .$$

[1] *Wied. Ann.*, **33**, 393, 1888.

[2] *Ibid.*, **50**, 293, 1893; also *Sitzungsberichte der Berliner Akademie*, 1893.

In the use of the slit-eyepiece, the incident and the diffracted rays coincided, that is

$$\delta = 2i .$$

Then the previous equation becomes:

$$m\lambda = 2c \sin i . \qquad (3)$$

which position corresponds to the minimum deviation.

The two methods of observation depending on equation (2) were exclusively used by me, particularly the slit-eyepiece. It required no setting on the direct image, which was impossible with my objective on account of parallax. I therefore in my measurements always set on the same order to the left and right, and took half of the resulting angle. The adjustment for the method of auto-collimation was the simplest imaginable. The axes of telescope and collimator coincided. After parallax had been eliminated between the slit and the green mercury line, the position of the grating was regulated by the two screws of the spectrometer table, until on turning the grating by the graduated circle (the telescope always remaining fixed by this method) the upper edges of the spectral images of the different orders, both left and right, all touched the upper edge of the slit as they passed through the field. Thus the rulings were parallel to their axis of rotation, that is, to the axis of rotation of the spectrometer table. Then the grating was slightly tilted by a very slight turning of the two screws of the spectrometer table, so that the spectral image reached over upon the slit, which could therefore be used as a pointer, and thus the adjustment was complete.

The adjustment of the spectrometer for using the collimator is decidedly more complicated, but the steps are familiar, and moreover were fully described by Kurlbaum (*loc. cit.*).

I first made a series of experiments for acquiring general acquaintance with the arrangement for auto-collimation.

The formula

$$m\lambda = 2c \sin i$$

shows that the spectral images of a definite wave-length of different orders must follow rapidly after each other, when we consider that the grating-constant is about 25 μ; in a theoretical case for $\sin i = 1$, even a 90th order should be possible for λ 5460. In these

experiments grating G_1 was superior to G_2 in the brightness of the images. Both gratings showed ghosts, but they were so faint that they did not in any case disturb the principal spectra. Both showed that the actual grating in no wise fulfils the ideal case, according to which the intensity of the spectra must decrease with increase of order. As I always used for my angular measurements two corresponding orders left and right, there were few of the great number of images present which could be used for my purpose, having an equal intensity right and left. With G_2 the 75th order was distinctly perceptible, while with G_1 I could go beyond the 80th. Aside from the lower orders, which I did not use on account of the too small angle, sufficient equality was found right and left for the following orders:

For G_1		For G_2	
Order	i (approx.)	Order	i (approx.)
20	12° 26′	22	13° 42′
39	24 50	41	26 13
53	34 49	72	50 52
65	44 27		
81	60 46		

I then made extensive experiments to determine whether the spectral lines of different high orders and of different orders right and left required a different focal setting; but, in spite of the sensitiveness of the objective, I was unable to detect any parallax in the different cases, a proof of the excellence of the gratings.

In the experiments with the use of the collimator tube, it was not possible, on account of the large diameters of the two tubes, to bring them closer than an angle of about 57°. Thus the lowest order which could be observed was the 38th; the 39th to the 42d orders could be used and were very bright.

THE DEFINITIVE MEASURES

I here used exclusively the mercury lines λ 5460 (green) and λ 5791 (yellow), giving up the comparatively weak line at λ 4358. I did not undertake a calibration of the graduated circle, but, instead of this, I at first turned the circle by 60° six times for each series of measures, and then repeated the measurement. The results

of this experiment soon justified me in contenting myself with a single repetition at 90°, so that the the effect of the eccentricity was eliminated. As is seen from the following table, I almost exclusively employed for the precise measures the method of auto-collimation, as this made it possible to work very quickly and surely.

I give now an example of a series of measures; all the measurements subsequently used follow this scheme. The atmospheric pressure was read from a barometer in an adjacent room. The readings in seconds given in the following scheme one above the other, which are afterward combined in a mean, correspond to each setting on the spectral line, and are in each case the mean of three settings of the double thread on the particular division of the circle concerned.

Grating G_1, *Hg* λ 5460. Auto-collimation. 65th order. Turned once 90°.
March 23, 1905

Circle in	Temp.	Pressure	65th Order	Microscope I		Microscope II	
				Separate Settings	Mean	Separate Settings	Mean
Position I	14°.76	729.0 at 14°.2 = 727.21 at 0°	left	18° 16′ 37″.1 37.0 37.6	37″.23	198° 13′ 53″.7 53.1 53.8	53″.53
	14.77		right	289 22 42.9 42.9 43.1	42.97	119 19 57.4 57.4 58.1	57.63
	14.78		left	18 16 38.1 38.4	38.25	198 13 54.0 54.4	54.25
Position II Turned 90	14.79	729.0 at 14°.3 = 727.20 at 0°	left	108° 16′ 34.0 34.2 34.0	34.07	288° 13′ 54.9 55.1 55.0	55.00
	14.79		right	19 22 40.0 39.7 39.3	39.67	209 19 57.9 57.2 57.4	57.50
	14.79		left	108 22 33.9 34.1	34.00	288 13 55.3 55.0	55.10

Position I: $2i = 88° 53'$ 54″.26, 55.90, 55.28, 56.62 → 55″.51₅

$i = 44° 26' 57''.76$ at 14°.77 and 727.21 mm pressure

Position II: $2i = 88° 53'$ 54″.40, 57.50, 54.30, 57.60 → 55″.96

$i = 44° 26' 57''.98$ at 14°.79 and 727.20 mm pressure.

I computed the wave-length λ from each of these two determinations separately, and I take the mean from the two values thus obtained as the definitive result of the series. The corrections applied in the following tables to the wave-lengths for the effect of temperature and pressure were reduced from the relation

$$\lambda\left(1+0.0000039\frac{p}{1+at}\right)=\text{constant}^{1}$$

where λ signifies a definite wave-length, p the pressure in millimeters, and t the temperature. Accordingly for $+1°$, λ increases by about $0.95\times10^{-6}\,\lambda$; and for -1 mm change of pressure, by $0.36\times10^{-6}\,\lambda$. The significance of the columns of the following tables, in so far as not directly stated, is as follows:

Two gives the line used; 3, the method of measuring with the spectrometer; 4, the order; 5, the number of repetitions; 6, the observed temperature (C), α being the mean and β the largest difference occurring during the measurement; 7, the air pressure reduced to 0°; 8, the angle of diffraction i, α being the mean of four differences, and β the largest difference in seconds; 9, the grating constant expressed in microns and reduced to the temperature of the angular measurement; 10, the wave-length λ for temperature and pressure of the angular measurement; 11, the corrections to λ on account of temperature and pressure for reduction to 20° and 760 mm; 12, λ plus reduction for the separate measures; 13, the final resulting λ for each series from the mean of the separate measures; 14, the residuals of the separate measures from this mean; 15, the residuals of each series from the final mean of all the series, hence from the definitive wave-length to be given below; 16 further gives the effect of the error of 1° C (α), 1 mm pressure (β), and under γ the effect of 1″ of angle of i or δ on the wave-length λ. All the quantities in columns 10 to 16 are expressed in Å.U.

The comparative measurement in the different orders for grating G_1 is more comprehensive than it could be by any earlier observer; it yielded a perfectly satisfactory agreement, all the differences falling within the limits of error mentioned. The same is to be expected for G_2, since the measures by auto-collimation and the measures with

[1] This is the formula which Kohlrausch gives (*Prakt. Phys.*, 1901, **9**, 588); it was satisfactory for my purposes, as I have proved.

the collimator tube, where the diffraction angle is about 2½ times greater in the latter case than in the former, also were in entire agreement. The final result of the spectrometer measurements, using the widths of the grating as determined by the Commission, is given in the tables as follows for 20° C. and 760 mm pressure.

	G_1	G_4	Mean
Green mercury line.....	5460.78_5	5460.75_3	5460.76_9
Yellow mercury line.....	5790.71_9	5790.67_9	5790.69_9

The inaccuracy of the results for each grating, on account of the errors of observation and of the instruments, can scarcely exceed a few hundredths of an Ångström unit for the line λ 5460; for λ 5791 it is doubtless larger, as I have contented myself with only one series of measures with that line. The object of this research came to be the measurement of one line accurately. As to the errors of observation and those of the graduated circle, I could probably have attained a certainty of approximately 0.01 Å. U. by increasing the number of spectrometer measurements; but I contented myself with the previous experiments, because, from what was said at the beginning of this paper, it would have been a fruitless undertaking to attempt to attain with gratings the accuracy of the results obtained by Messrs. Perot and Fabry with the interferometer. The question was rather this with respect to the results of Kurlbaum and Bell: Will my results agree among themselves, and with those of Perot and Fabry within the limits set by the errors of observation, of the graduated circle, and of the comparator? We see that my gratings exhibited between themselves differences similar to those in the gratings used by Bell; compared with the value found by Perot and Fabry at 20° and 760 mm for the green line, viz. 5460.768 Å. U., my results showed the following differences:

Pérot and Fabry $-G_1 = -0.01_7$ Å. U.
" " " $-G_2 = +0.01_5$ "

The differences of the results of Kurlbaum and of Bell, respectively, from those of Perot and Fabry, for D_1 (λ 5896) are

Kurlbaum – Pérot and Fabry: −0.10 and +0.02 Å. U.
Bell– " " " +0.22 " +0.27 "
" " " " +0.19 " +0.21 "

Gratin

1 Date 1905	2	3	4	5	6		7	8 Angle of Diffraction		9 Grating Constant
					α	β	b	α	β	
22, III	*Hg* 5460 green	Auto-collimation	20	Twice by 90°	19°.16	0°.06	730.37	12° 26′ 31″.54	1″.0	25.3458$_0$
					17.71	0.00	727.53	33.00	0.3	51$_3$
22, III	"	"	39	"	19.00	0.01	730.37	24 50 35.51	2.6	25.3457$_2$
					18.95	0.06	730.20	35.18	1.6	57$_0$
9, II and 10, II	"	"	53	Six times each by 60°	16.03	0.09	744.53	34 49 7.88	2.3	25.34434
					14.14	0.14	744.30	12.08	3.8	34$_7$
					17.29	0.01	739.87	2.77	3.8	49$_2$
					17.30	0.01	739.84	3.86	1.5	49$_3$
					17.09	0.02	739.74	5.47	1.9	48$_3$
					17.07	0.04	739.74	3.64	2.2	48$_1$
13, II	"	"	53	"	16.05	0.12	742.18	34 49 7.62	1.1	25.34434
					16.18	0.04	742.01	4.81	1.3	44$_0$
					16.20	0.01	742.10	5.91	0.5	44$_1$
					16.22	0.04	742.08	5.57	1.5	44$_2$
					16.22	0.04	742.31	5.63	2.5	44$_2$
					16.22	0.03	742.22	5.81	3.5	44$_2$
23, III	"	"	65	Twice by 90°	14.77	0.02	727.21	44 26 57.76	1.2	25.34376
					14.79	0.00	727.20	57.98	1.6	37$_7$
25, III	"	"	81	"	16.29	0.01	730.29	60 45 57.29	4.8	25.34446
					16.37	0.06	730.20	56.94	1.3	44$_9$
14, II	5791 yellow	"	53	Six times each by 60°	16.14	0.08	742.98	37 15 48.32	1.3	25.34430
					16.17	0.05	742.97	48.01	3.0	44$_1$
					16.16	0.05	742.95	47.37	3.4	44$_0$
					16.16	0.06	742.85	46.27	2.2	44$_0$
					16.05	0.07	742.89	48.87	3.4	43$_4$
					16.02	0.05	742.93	46.06	1.2	43$_3$

Gratin

21, II	*Hg* 5460 green	Auto-collimation	41	Twice by 90°	12.96	0.09	730.36	26° 12′ 50″.30	0″.9	25.34278
					12.98	0.04	730.65	51.64	0.9	27$_9$
21, II	"	"	41	"	21.67	0.04	731.36	26 12 35.69	1.5	25.3468$_1$
					21.59	0.05	732.25	36.39	0.4	67$_7$
17, III	"	Grating perpendicular. Coll. tube Slit 0.06 mm wide	41	"	14.05	0.12	727.88	62 3 35.69	13.8	25.34328
					14.39	0.10	727.88	33.90	15.0	34$_4$
27, III	"	Auto-collimation	41	"	11.36	0.10	733.96	26 12 53.99	1.0	25.3420$_9$
					11.60	0.07	733.75	52.37	1.2	21$_5$
28, III	"	"	41	"	22.84	0.02	734.41	26 12 32.42	1.3	25.34736
					22.82	0.04	734.40	32.90	2.0	73$_6$
22, II	*Hg* 5791 yellow	"	41	"	16.13	0.04	728.93	27 55 47.38	1.2	25.3442$_4$
					16.11	0.01	728.83	47.23	3.3	42$_3$

Grating G_1

9 Grating Constant c	10 $\lambda_{l,b}$	11 Corrections l	11 Corrections b	12 λ	13 λ Mean	14	15	16 Effect of Errors: 1°	16 Effect of Errors: 1 mm	16 Effect of Errors: 1″
$25\cdot3458_0$	5460.82_8	$+0.0044$	-0.0583	5460.76_9	5460.78_6	∓0.017	$+0.001$	on the grating constant 4.6 Å. U.; hence on λ: 0.098 Å. U.	0.002	0.107
51_3	85_6	$+0.0119$	-0.0638	80_3						
$25\cdot3457_2$	5460.85_3	$+0.0052$	-0.0583	5460.80_0	5460.78_7	∓0.013	$+0.002$		0.002	0.057
57_0	82_7	$+0.0054$	-0.0586	77_4						
$25\cdot3443_4$	5460.83_3	$+0.0206$	-0.0304	5460.82_3		$+0.035$				
34_7	80_5	$+0.0304$	-0.0309	80_5		$+0.017$				
49_2	76_5	$+0.0141$	-0.0306	73_6	5460.78_8	-0.049	$+0.009$		0.002	0.038
49_3	80_6	$+0.0140$	-0.0306	78_9		$+0.001$				
48_3	84_7	$+0.0151$	-0.0308	82_2		$+0.034$				
48_1	77_5	$+0.0152$	-0.0308	75_1		-0.037				
$25\cdot3443_4$	5460.82_3	$+0.0205$	-0.0350	5460.80_9		$+0.053$				
44_0	72_9	$+0.0198$	-0.0353	71_3		-0.043				
44_1	77_3	$+0.0197$	-0.0352	75_7	5460.75_6	$+0.001$	-0.029		0.002	0.038
44_2	76_3	$+0.0196$	-0.0352	74_7		-0.009				
44_2	76_4	$+0.0196$	-0.0348	74_9		-0.007				
44_2	77_1	$+0.0196$	-0.0328	75_8		$+0.002$				
$25\cdot3437_6$	5460.83_0	$+0.0271$	-0.0645	5460.79_3	5460.79_7	∓0.004	$+0.012$		0.002	0.027
37_7	83_8	$+0.0270$	-0.0645	80_1						
$25\cdot3444_6$	5460.83_5	$+0.0193$	-0.0584	5460.79_6	5460.79_6	±0.000	$+0.011$		0.002	0.015
44_0	83_6	$+0.0188$	-0.0586	79_6						
$25\cdot3443_0$	5790.76_7	$+0.0212$	-0.0355	5790.75_3		$+0.034$				
44_1	76_0	$+0.0211$	-0.0355	74_6		$+0.027$				
44_0	73_5	$+0.0211$	-0.0355	72_1	5790.71_9	$+0.002$	$-$	on λ: 0.105 Å. U.	0.002	0.035
44_0	69_3	$+0.0211$	-0.0358	67_8		-0.041				
43_4	77_6	$+0.0217$	-0.0357	76_2		$+0.043$				
43_3	67_0	$+0.0219$	-0.0356	65_6		-0.063				

Grating G_2

$25\cdot3427_8$	5460.74_5	$+0.0365$	-0.0582	5460.72_3	5460.76_0	∓0.037	$+0.007$	on the grating-constant: 4.6 Å. U.; hence on λ 0.098	0.002	0.0535
27_0	81_9	$+0.0364$	-0.0577	79_7						
$25\cdot3468_1$	5460.82_9	-0.0087	-0.0563	5460.76_5	5460.78_0	∓0.015	$+0.027$		0.002	0.0535
67_7	85_8	-0.0083	-0.0546	79_5						
$25\cdot3432_8$	5460.78_3	$+0.0309$	-0.0632	5460.75_1	5460.75_5	∓0.004	$+0.002$		0.002	0.014
34_4	79_3	$+0.0291$	-0.0632	75_9						
$25\cdot3420_9$	5460.79_6	$+0.0448$	-0.0512	5460.79_0	5460.75_3	±0.037	-0.000		0.002	0.0535
21_5	72_4	$+0.0436$	-0.0516	71_6						
$25\cdot3473_6$	5460.77_1	-0.0147	-0.0503	5460.70_6	5460.71_8	∓0.012	-0.035		0.002	0.0535
73_6	79_5	-0.0146	-0.0503	73_0						
$25\cdot3442_4$	5790.72_4	$+0.0213$	-0.0611	5790.68_4	5790.67_0	±0.005	$-$	on λ: 0.105	0.002	0.053
42_3	71_4	$+0.0214$	-0.0613	67_4						

Hence we see that my results are in thoroughly satisfactory agreement with those of Pérot and Fabry within the limits of accuracy. Kurlbaum's results approach this agreement, but it must be admitted that, while Bell's experiments were carried out with the same care, and furthermore with better gratings than Kurlbaum's, nevertheless, or perhaps on that account, the above comparison compels us to assume that Bell was subject to an error which cannot be attributed to the defects in his gratings, as treated by me in Sections II and III.

The question now is: How do my results compare with the conclusions of Section III? We must here bear in mind that the calibration only partially explains the action of the whole grating, inasmuch as the larger intervals of the grooves (hence from o—3119 or o—3084 rulings) are effective for the whole grating, and these were not investigated in the calibration. This explains why the line given in the table on page 250 as "center of gravity" line need not fall in coincidence with the spectral image of the entire grating. My experiments with the oscillations, however, give a basis for the following considerations, which are due to Privatdozent Dr. R. Gans, of Tübingen, and the computations based upon them.

The theory of the grating gives for the distribution of intensity of the diffraction pattern the following relation.[1]

$$J_o = J'_o \frac{\sin^2 \frac{\mu a}{2}}{\left(\frac{\mu a}{2}\right)^2} \cdot \frac{\sin^2 \frac{\mu(a+d)}{2} p}{\sin^2 \frac{\mu(a+d)}{2}}, \qquad (1)$$

in which

$$\mu = \frac{2\pi}{\lambda}(\sin \phi + \sin \psi), \qquad (2)$$

where λ denotes the wave-length, ϕ the angle of incidence, and ψ the angle of diffraction. J'_o denotes the intensity for the diffraction angle zero ($\mu = o$), as it would be for a slit; p is the number of rulings, a the reflecting portion between two rulings, and d the width of a ruling, whence $a+d$ is the grating constant.

The intensity regarded as a function of μ is a maximum, if

$$\frac{dJ_o}{d\mu} = o. \qquad (3)$$

[1] See Mann and Millikan's translation of Drude's *Optics*, 1900, p. 222.

This occurs for

$$\mu_0=\frac{2\pi}{a+d}m \quad (m=0,\ 1,\ 2\ldots), \tag{4}$$

m being the order of the diffraction image.

Now, if there is a small error of any given sort in the grating, the intensity of the diffraction pattern will no longer be given by J_0 from equation (1), but it will be

$$J=J_0+J_1, \tag{5}$$

where J_1 is a small quantity to be added to J_0, and is also a function of μ.

The position of maxima is now found from

$$\frac{dJ}{d\mu}=\frac{dJ_0}{d\mu}+\frac{dJ_1}{d\mu}=0. \tag{6}$$

This equation is no longer satisfied by μ_0 from (4) but by

$$\mu=\mu_0+\delta\mu, \tag{7}$$

where $\delta\mu$ indicates the small error in position of the diffraction pattern. Neglecting the higher powers of $\delta\mu$, (6) and (7) give

$$\left(\frac{dJ_0}{d\mu}\right)_{\mu=\mu_0}+\left(\frac{d^2J_0}{d\mu_2}\right)_{\mu=\mu_0}\delta\mu+\left(\frac{dJ_1}{d\mu}\right)_{\mu=\mu_0}=0. \tag{8}$$

From (3) and (4) the first term of (8) disappears, and we obtain for the error in position of the diffraction pattern

$$\delta\mu=-\left(\frac{\frac{dJ_1}{d\mu^2}}{\frac{d^2J_0}{d\mu^2}}\right)_{\mu=\mu_0}. \tag{9}$$

Consider now the amplitude. For a grating without error, let the amplitude of the diffraction pattern in a definite order be expressed by A_0, if only the nth part of the grating is effective. If the whole grating is effective, so that there are n elements of the wave in the same phase, the amplitude will be $n\mathrm{A}_0$. Correspondingly the intensities $J_0=A_0^2$ or $J_0=n^2A_0^2$, according as the nth portion or the whole grating is effective. This may also be derived from (1).

For an imperfect grating the amplitude is given by

$$A=A_0+A_1, \tag{10}$$

in case the nth part of the grating is effective. A_1 is here a small quantity in comparison with A_0. If the whole grating is used, the amplitude becomes

$$A = nA_0 + \sum A_1 . \tag{11}$$

$\sum A_1$ denotes the sum of the amplitudes, which are added on account of the errors of the whole grating. But in case the errors are not of a systematic nature[1] the quantities A_1, on account of the different phases of the erroneous elements, will regularly alternate in quantity, magnitude and sign, so that $\sum A_1$ is of the order of A_1 and not of the order nA_1.

The intensity of the nth portion of the erroneous grating is therefore

$$A^2 = A_0^2 + 2AA_1 , \tag{12}$$

since we may neglect A_1^2.

But the intensity of the whole grating is

$$\left.\begin{aligned} J = J_0 + J_1 &= \left(nA_0 + \sum A_1\right)^2 \\ &= n^2 A_0^2 + 2nA_0 \sum A_1 \end{aligned}\right\} . \tag{13}$$

Therefore

$$J_1 = 2nA_0 \sum A_1 . \tag{14}$$

The numerator of (9) will therefore be of an order n times as great if the whole grating is used instead of its nth portion, while, on the contrary, the denominator will become n^2 times as large. It follows from this that the order of magnitude of the error in the position of the diffraction image becomes n times smaller if the whole grating acts, than if only its nth portion acts.

This result for the error is not rigorously correct, since A_0 and A_1 depend upon the whole grating-width in the manner given, while the differential quotients with respect to μ depend upon it in a complicated manner, and, moreover, perhaps not only the higher orders of $\delta\mu$ may be neglected. In any case (9), in connection with the reasoning regarding the amplitude, gives an idea of the error committed.

[1] See Lord Rayleigh, *Phil. Mag.* (4) **47**, 193–205, 1874; *Encyc. Brittanica* (9) **24**, 438.

Since my gratings, as has been shown, satisfy the conditions that no systematic errors are present, what precedes can be immediately applied to Section III. I give here as there, for the whole grating also, the order of the deviations in seconds, in the last column.

Grating	Desig. of Experiment	Angle of Diffraction i	Effect in Å. U. of Error of 1″	n	Greatest Departure from "Center" Line	Order of Magnitude of Deviations for Entire Grating
G_1	A	26° 1′	0.054	$\frac{3\,1\,1\,9}{4\,0}=78.0$	19″.0	0″.24
G_1	C*	26 1	0.054	$\frac{3\,1\,1\,9}{2\,0\,0}=15.5$	5.1	0.34
G_1	D	26 1	0.054	$\frac{3\,1\,1\,9}{2\,0\,0}=15.5$	5.6	0.36
G_1	E	26 34	0.053	$\frac{3\,1\,1\,9}{2\,0\,0}=15.5$	6.4	0.41
G_2	F	19 34	0.074	$\frac{3\,0\,8\,4}{2\,0\,0}=15.4$	5.3	0.34
G_2	G	19 35	0.074	$\frac{3\,0\,8\,4}{1\,2\,0}=25.7$	7.9	0.31

* See table, p. 348.

The effect of the error on λ according to this table would therefore be in an extreme case

For grating G_1, 0.022 Å. U.
" " G_2, 0.025 " .

This explains the excellence of my results on page 258 ff.

CORRECTION[1]

Dr. Otto Schönrock, in Berlin, has had the kindness to call my attention to an error in my computation of the curvature of the grating surface (Section II). In Fig. 1 I incorrectly gave for the numerical value for the length A_1C the largest possible departure from the plane, or $\lambda/4$ and $\lambda/2$. But the correct reasoning is as follows, according to Brodhun and Schönrock.[2] If in Fig. 1 we call $NA_1=NA_2=R$ the radius of curvature of the grating A_1A_2, assumed to be concave, and d the versed-sine of the arc A_1A_2; then

$$R=\frac{b^2}{8d}+\frac{d}{2}$$

[1] *Annalen der Physik*, **22**, 798, 1907.

[2] *Zeitschrift für Instrumentenkunde*, **22**, 353, 1902.

or, since $d/2$ vanishes as compared with R,

$$R=\frac{b^2}{8d}.$$

A simple consideration of the figure shows that

$$\gamma=\frac{b}{R}=\frac{8d}{b}.$$

Mr. Schönrock also believes, on the basis of his experience, that it is safe to infer from my observations that for each of the two gratings $d<\lambda/2$; but that it is not at all impossible that d may be nearly equal to $\lambda/2$ for each grating. Hence, in the extreme case, $d=\lambda/2$ $=0.000295$ mm. It is further to be considered that the objective used in the experiment with Fizeau's fringes had an aperture of only 70 mm; hence the radius of curvature will be

$$R=\frac{70^2}{8\cdot 0.000295}=2.08 \text{ km}.$$

For $b=78$ mm, γ becomes $7''.7$, and is thus considerably larger than as computed by me as $1\frac{3}{4}''$.

With the apparatus described by Brodhun and Schönrock it is furthermore possible, by varying the inclination of the plates producing the interference, to follow the wandering of the fringes so as to derive an idea ultimately as to the nature of the curvature. My improvised apparatus did not permit of any such manipulation.

While I must admit from what precedes that the second section of my paper is defective, I should like to emphasize that this has slight bearing on the essential part of the research. For on the one hand, nothing was known by me as to the character of the quartz plate used. In Mr. Schönrock's opinion the plane surfaces by Brashear are quite as perfect as, if not more so than, the quartz surface by Zeiss. On the other hand, the calibration described in Section III adequately cares for the errors of curvature in practice. They can indeed be only very insignificant, inasmuch as the different high orders exhibited no focal errors.

A PHOTOGRAPHIC METHOD FOR THE DETECTION OF VARIABILITY IN ASTEROIDS

By JOEL H. METCALF

By the method described in the *Astrophysical Journal* for May 1906 (**23**, 306–311) for the discovery of asteroids, the photographic plate is made to follow the mean motion of asteroids in opposition at that season of the year in which the plate is taken. As a result the stars are drawn out into trails, the length of which depends upon the exposure time. The asteroids, on the other hand, are points like the ordinary star images of chart-plates.

In long exposures any departure of the actual asteroid from the mean motion will show itself in the elongation of the images. Practically most asteroids can be followed for 35 or 40 minutes without this departure becoming apparent.

As it would be easy to mistake an imperfection on a plate for an asteroid, two exposures are always taken, separated by a small arbitrary amount in right ascension. Each asteroid is therefore represented by two more or less circular images in close proximity.

If the time is equal in both exposures, the star-trails will be of the same length, and the two images of the asteroid ought to be of the same size, shape, and density.

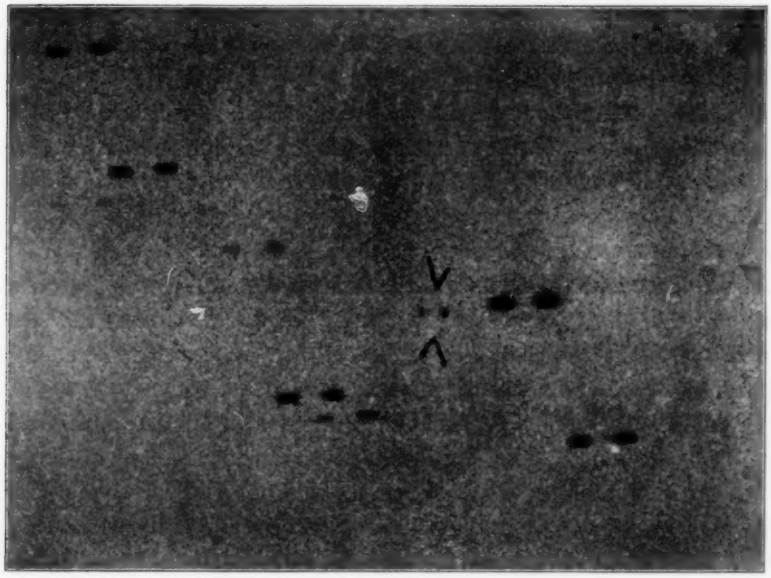
Practically this is not always true, for two reasons: first, the motion of the photographic plate may not have been uniform during the exposure; second, there may have been a change in the transparency of the atmosphere between the first and second exposures due to changing altitude of the asteroids or to a gathering storm.

In both cases, however, the star-trails would register this change. If the motion of the plate were not uniform, the star-trails would not be straight and of similar length. If the transparency of the air changed, there would be differences shown in the density of the fainter star-trails.

Now, if on any plate the star-trails are of the same length and density, showing that the motion of the plate has been uniform and that the atmosphere has not changed, then, if the images of the asteroid

are not of the same size and density, the presumption is that there has been a change in the luminosity of the asteroid.

The only escape from this would be that the photographic plate contained a defect or was of unequal sensitiveness. Defects are possible, but are usually apparent under a high magnifying power. I think the experience of the discoverers of variable stars is against a belief in any great differences of sensitiveness in such close proximity as the adjacent images of an asteroid (about 0.3 mm on my plates).



I therefore believe that any plate fulfilling the above conditions as to atmosphere and uniform motion, and yet showing the asteroid images to be greatly different in size and density, would establish a probable variability.

By the usual method of picking up asteroids the star images are points and the asteroids trail. If a plate was found in which the trail showed greater density in one part than another, there would be no check by which it could be decided whether it was due to variability or the hazing of the atmosphere. Other asteroids even on the same plate would not be a test, for often local cloudiness is apparent even

on the best nights in our climate. If the stars trail, however, they become an infallible index of these changes.

That these remarks are not entirely theoretical I would contend from the accompanying illustration. It is an enlargement of a photograph made on November 6, 1906, of *1906 WE*, discovered at Taunton on October 26, 1906. I believe the star-trails show that the motion of the plate was practically uniform and that there was between the exposures no great change in the transparency of the atmosphere; and yet the two images of the asteroid show a surprising difference in size and density. Unfortunately, due to an accident, this asteroid was observed only four times—on October 26, November 6, December 7, and December 18, 1906.

On every plate there are two images of the asteroid, and yet on three of them—those of October 26, November 6, and December 18—the companion images show differences of size and density, although the differences on October 26 and December 18 are not so marked as on November 6.

On the plate of November 6 the exposure times were 35 minutes each, with 1 minute between exposures. If this change is real and not illusory, it shows that *1906 WE* went through a noticeable change of brightness in 1 hour and 11 minutes.

I believe this method of photographing asteroids, containing as it does such a good check on atmospheric changes, might be extended much farther. A long series of images might be taken, not only on one night, but on several nights, in a climate where good weather prevails, and the corresponding star-trails would in every case be a check on the condition of the atmosphere. If there arose any difficulty in getting absolute magnitudes from the trouble of comparing images with trails, the motion of the plate might be made just half the real motion, and then the asteroid and stars would be of equal length, and there would only be the difficulty of identifying the asteroid trail.

For variations taking place, however, in the course of a few hours, as many circular images as possible should be taken, and the exposure time should be as short as is consistent with obtaining a good image of the particular asteroid to be observed.

TAUNTON, MASS.
April 8, 1907

ON THE STRUCTURE OF THE FINEST SPECTRAL LINES[1]

By O. von BAEYER

In a previous paper E. Gehrcke and O. von Baeyer communicated their observations on the resolution of different spectral lines by the method of interference points, giving at the same time quantitative measures of the distance of the satellites from the principal lines concerned.[2] Meantime the Reichsanstalt purchased from A. Hilger, of London, a plate surpassing in its excellence all of the plates hitherto acquired by the Physikalisch-Technische Reichsanstalt. Investigations on the Zeeman effect in faint magnetic fields were promptly begun with this plate, and reference made to the excellent qualities of the plate in the publication.[3] It therefore seemed desirable to repeat with this improved apparatus the measurements on the most important spectral lines which had thus far been proven to have satellites. The results thus obtained are here communicated.

I. ARRANGEMENT OF EXPERIMENT

The arrangement is entirely similar to that described in the previous paper. For producing the interference points, the Hilger plate was crossed with one of 3 mm thickness by Haecke of Berlin, which had been employed in our previous work. It is identical with the plate[4] designated in the other paper as D, of 0.317 cm thickness, 15 cm length, and of refractive index 1.53. The plate by Hilger is 0.971 cm thick, 30 cm long, 4 cm high, and has a refractive index of 1.58. It exhibits over its whole surface variations of thickness which may be estimated as only one-twentieth of the wave-length 436 $\mu\mu$. In

[1] A communication from the Physikalisch-Technische Reichsanstalt. Translated from the *Verhandlungen der Deutschen Physikalischen Gesellschaft*, **9**, No. 4, 1907.

[2] *Annalen der Physik*, **20**, 269, 1906.

[3] E. Gehrcke and O. von Baeyer, *Verhandlungen der Deutschen Physikalischen Gesellschaft*, **8**, 399, 1906; and *Physikalische Zeitschrift*, **7**, 905, 1906.

[4] This plate was kindly put at the disposal of the Reichsanstalt by the firm of Haecke, of Berlin. It was meanwhile purchased by Professor M. Wien, of Danzig, but was kindly left with us until the completion of these measurements. I would not fail to express to Professor Wien my best thanks for this favor.

consequence of the great breadth and uniformity of the plate, a far greater brightness could be given to the phenomenon, and thus the exposure time of the photographs was considerably diminished. In general, about one-half of the previous exposure time was sufficient.

The revolving power, i. e., the quantity $\frac{\lambda}{\delta\lambda}$, where $\delta\lambda$ is the least difference of wave-length resolvable with the apparatus, is for this plate in round numbers 700,000 (for $\lambda = 0.5\ \mu$). The plate formerly used, C, from Zeiss (5 mm), had a resolving power of about 400,000. The echelon which Mr. Janicki[1] used had a resolving power of 420,000 (for $\lambda = 0.5\ \mu$).

II. MEASUREMENTS

a) *Spectrum of mercury.*—For producing this light, the Arons mercury lamp of Lummer type again served in part. For the yellow and green mercury lines the sharpness attained is entirely sufficient. A mercury vapor lamp by Boas, of Berlin, such as is used for technical purposes of illumination, did not give the same degree of brightness, and even the green line was not as sharp as could be desired. For the blue and violet lines a Geissler tube with mercury electrodes had to be used. Even in the Geissler tube the method of excitation had a great effect on the sharpness of the lines. Thus, for instance, in using a mercury interrupter by Boas, the lines at 434.8 $\mu\mu$ obtained on gentle warming of the Geissler tube were very diffuse; while a platinum interrupter, also by Boas, which furnished a very bright mercury spectrum in the same Geissler tube without exterior heating and with the same induction coil, yielded sharp lines. It would therefore seem as if the increase of the pressure of the mercury vapor by external heating broadened the lines more readily than the increase in temperature produced by the greater density of current.

The results of the measures agree in general with the previous values,[2] as the following table shows.

[1] *Annalen der Physik*, **19**, 36, 1906.

[2] E. Gehrcke and O. von Baeyer, *Annalen der Physik*, **20**, 269, 1906; L. Janicki, *ibid.*, **19**, 36, 1906. Observations on the positions of the satellites of the mercury lines have also been made by Pérot and Fabry, Prince Galitzin, and F. Bates, which are not cited further in this communication.

λ (μμ)	10^{-3} μμ		I	Echelon. Janicki	I
	Plate C and D Gehrcke and v. Baeyer	Hilger Plate and Plate D. v. Baeyer			
404.7	−1.13	−1.26	4	−1.11	$\frac{1}{8}$
	−0.49	−0.50	1	−0.51	$\frac{2}{3}$
	+0.77	+0.75	3	+0.67	$\frac{1}{2}$
	+1.38	+1.43	2		
407.8 weak	−0.77	−0.86	2	−0.76	$\frac{1}{6}$
	−0.44	−0.53	2	−0.46	$\frac{1}{8}$
		+0.37	4	+0.32	$\frac{1}{10}$
	+0.58	+0.55	1	+0.49	$\frac{1}{4}$
	+0.89	+0.82	3	+0.74	$\frac{1}{9}$
433.9	No satellite visible			−1.2	$\frac{1}{10}$
				+0.6	$\frac{1}{10}$
434.8 weak	−0.48	−0.54	1	−0.46	$\frac{1}{5}$
	+0.55	+0.56	3	+0.53	$\frac{1}{7}$
	+0.82	+0.89	2	+0.83	$\frac{1}{7}$
435.9	−1.71	−1.75	2		
	−1.18	−1.18	4	−1.12	$\frac{2}{3}$
	−1.08	−1.03	4	−0.97	$\frac{2}{3}$
		−0.48	7	−0.52	$\frac{1}{7}$
	−0.17	−0.18	1	−0.23	1
		+0.21	5	+0.20	$\frac{1}{5}$
	+0.27	+0.30	6		
	+0.51	+0.49	5	+0.43	$\frac{1}{4}$
				+1.05	$\frac{2}{3}$
	+1.21	+1.26	5	+1.2	$\frac{2}{3}$
	+2.02	+2.04	3		
491.6	No satellite visible				
546.1	−2.41	−2.50	2	−2.32	$\frac{1}{2}$
	−1.03	−1.07	4	−0.99	$\frac{1}{10}$
		−0.72	3		$\frac{1}{7}$
	−0.55	−0.51	5	−0.66	
		−0.25	1		
	+0.93	+0.87	1	+0.88	$\frac{1}{8}$
	+1.40	+1.32	3	+1.33	$\frac{1}{8}$
	+2...?	+2.22	6		
576.9				−1.13	$\frac{1}{8}$
	−0.56	−0.50	2	−0.50	$\frac{1}{8}$
	+0.47	+0.46	1	+0.46	$\frac{1}{8}$
				+0.87	$\frac{1}{10}$
				+1.20	$\frac{1}{10}$
579.0				−2.51	$\frac{1}{10}$
	−2.02	−1.9...	4	−1.87	$\frac{1}{10}$
	−1.28	−1.27	1	−1.19	$\frac{1}{8}$
				+0.84	$\frac{1}{6}$
	+1.43	+1.39	2	+1.32	$\frac{1}{5}$
				+1.68	$\frac{1}{10}$
	+2.41	+2.37	3	+2.30	$\frac{1}{7}$

Attention should be called in detail to the following points:

Hg λ 407.8. The satellite $+0.37.10^{-2}$ $\mu\mu$ occurs, which was previously observed by Janicki. λ 434.4 shows no satellite, in spite of long exposure (4 hours).

λ 435.9 shows, in comparison with the earlier results of Gehrcke and von Baeyer, two new satellites. One at −0.48 was also already observed by Janicki. The other arises from the splitting of the former satellite +0.27, which first became visible with the high resolving power. In accordance with the earlier measurements of Gehrcke and von Baeyer, the satellite at −1.75 appeared, which

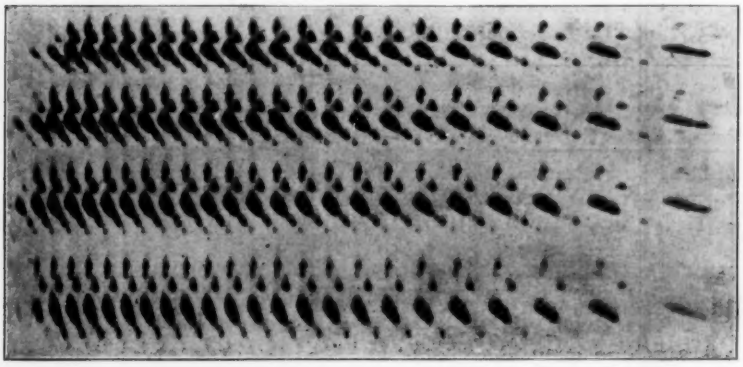


FIG. 1

Janicki did not correctly assign, and therefore considered to be a line at +1.05.

λ 546 $\mu\mu$ gave three new satellites. Fig. 1 shows an enlargement of this plate on which the satellites can be recognized, excepting the one faint one of intensity 6. In order to make the position of the satellites distinct, Fig. 2 gives a schematic representation of a principal line and its associated satellites. The numbers correspond to the intensities. See also the table.

We see at first that the earlier satellite −0.55 is split into two: −0.72 (intensity 3) and −0.51 (intensity 5). This also explains the uncertainty in the measurement which hitherto applied to this satellite. The principal maximum has in its immediate neighborhood a very strong satellite −0.25 (intensity 1). +2.22, which is exceedingly faint, is also new. This was not included in the earlier work of

Gehrcke and von Baeyer, although it was noticed; but, as it was then hardly visible, and moreover fell very close to a principal maximum, we regarded it as false. But the photographs with the Hilger plate show it very clearly. If Janicki did not observe it, this was due to the fact that on his photographs this satellite coincided with -2.50. The distance of the two principal maxima at this point is 0.476 Ångström units, so that -2.50 and $+2.22$ were only separated by 0.004 Å. U., which his echelon could no longer resolve.

In this case the advantage is very clearly apparent which the method of interference points has over the method of interference bands, for with observations with the Hilger plate only, as may be seen from Fig. 1, the strong satellites with intensities 1, 1, 2, 3, would have coalesced with the principal line to form a single, vertical, greatly broadened band.

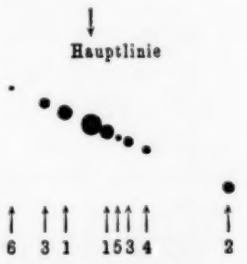


FIG. 2.—*Hg* 546.1 $\mu\mu$ with 10 mm plate by Hilger and 3 plate (D) by by Haecke.

At λ 576.9 and 579.0, the fainter satellites given by Janicki again failed to appear.

As to λ 579.0, it is to be remarked that satellite -2.02, which is only faintly indicated, could not be accurately measured. On this plate there also appears a pretty strong line which could not be a ghost. On the other hand, it could not be included among the satellites, as it did not lie in the direction of the remaining satellites. This is evidently a widely distant line. It can only be said, on the basis of the measures, that its distance is in any case more than $4 \cdot 10^{-2}$ $\mu\mu$. It therefore ought to be possible to observe it with a good grating. The possibility is moreover not excluded that it is the already known line λ 580.4.[1]

b) *Bismuth.*—The bismuth amalgam lamp of quartz, previously described, was used for the production of the spectrum of bismuth. The line λ 472.2 was investigated and yielded results in perfect accordance with the former measures.

From the good agreement of the earlier measures and those communicated here, it is again to be inferred that the method of inter-

[1] Kayser and Runge, *Abhandlungen der Berliner Akademie der Wissenschaften*, 1891.

ference points, even with the use of less perfect plates, such as Plate C, yields unambiguous results, and renders it possible to distinguish ghosts from the true satellites. Every step forward in the production of plane-parallel plates will also further diminish the only disadvantage

	Gehrcke and von Baeyer, Plate C and D	von Baeyer, Hilger Plate and Plate D	I
	+0.66	0.63	1
	+1.12	1.08	3
472.2 μμ	+2.61	2.63	2
	+3.07	3.07	1
	+3.36	3.45	1

of the method, namely, the faintness of the phenomenon; and it is to be expected that in this point also the performance of the echelon will be at least reached.

As to the splitting of the spectral lines, the above investigation shows that the number of the certainly proven satellites has again increased, and it seems probable that every increase in the resolving power will always bring further new satellites into visibility.

ON THE BRIGHTNESS OF THE INNER EDGE OF THE PENUMBRA IN SUN-SPOTS (SECOND NOTE)

By S. CHEVALIER

Some time ago I wrote a note to show that in photographs made at the Zô-sè Observatory the inner edge of the penumbra of sun-spots was very generally brightened, just as it appears to be when looked at in a good telescope. Since this note was written, improvements have fortunately been made at this observatory in photographing the sun. I consequently feel obliged to return to the subject.

The photographs of the sun, taken daily, as far as possible, at Zô-sè, vary in diameter from 63 to 65 mm. When atmospheric circumstances are favorable, they show distinctly the small granules of the photosphere, but fail to represent all the details of sun-spots. Hundreds of features observed at the eyepiece and traced on drawings are wanting on the photographs. This is certainly due, in large part, to the smallness of the picture. The nucleus especially, unless crossed by some brilliant "bridge" or filled with only half-obscure material, appears quite dark. All around the nucleus there is a very faint ring, visible only with the greatest difficulty. Outside of this ring begins the inner edge of the penumbra, which looks very generally brighter than the outer one. This is the fact which in my first note I tried to set forth as a common occurrence in sun-spots.

But I have now strong evidence that this brighter ring which skirts the umbra in these photographs is not really the actual edge of the umbra. Within the brighter ring there is a fainter one, just mentioned, probably consisting of the sharp ends of the filaments which run still farther inside the nucleus and which must be considered as belonging to the penumbra. So that what appears on our ordinary photographs, taken at Zô-sè, as the edge of the penumbra, is as a matter of fact at some distance from the nucleus. On ordinary photographs the nucleus always comes out much larger than it ought to and obscures the very minute details along the brim.

Before inquiring into the cause, the fact must be clearly demonstrated. For this purpose, among many instances, I will select two.

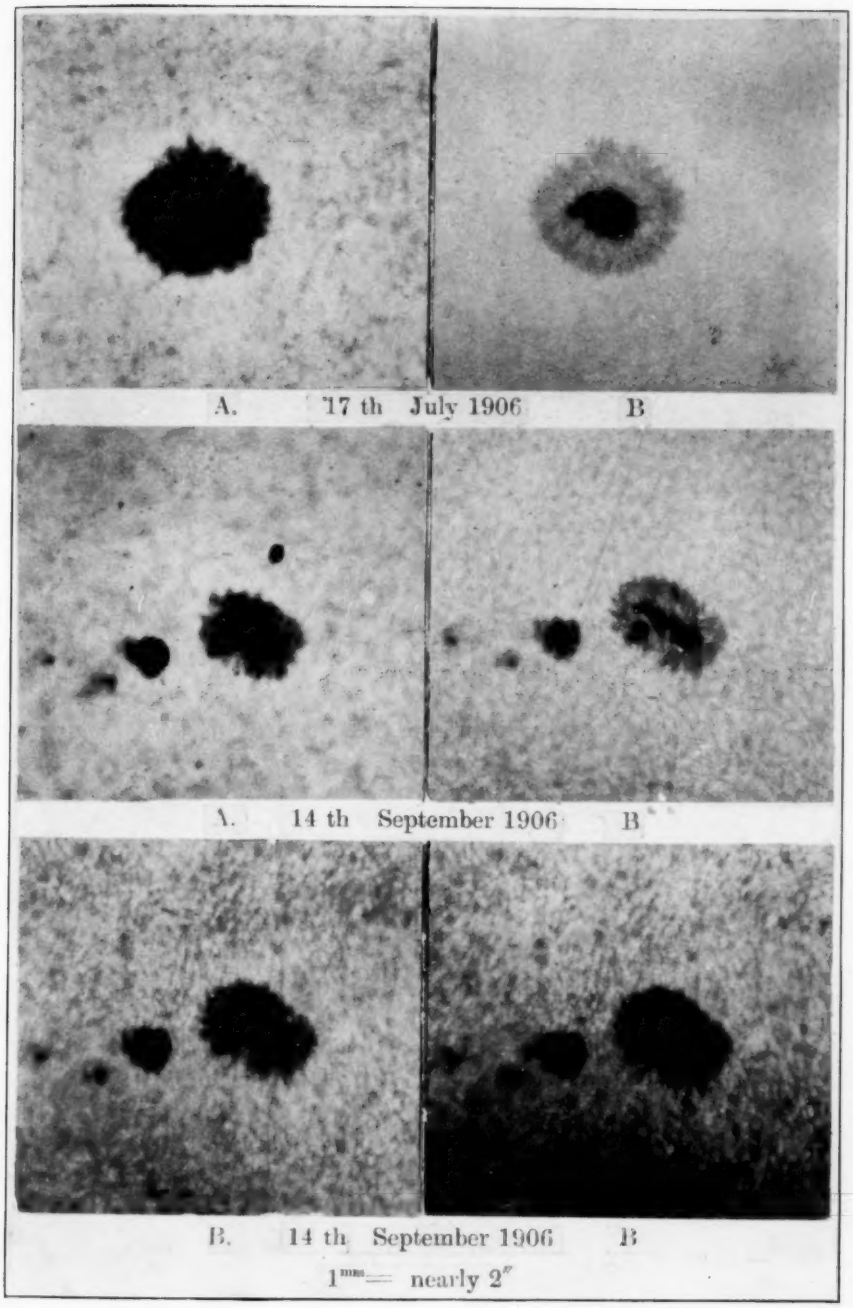
In each case I will put before the reader two images of the same sun-spot. One, which for the sake of brevity I will hereafter call *A*, is an enlargement to 16 diameters of a photograph made in the ordinary way, viz., on a plate at the focus of the telescope. The other, which I will call *B*, is a photograph obtained by the direct enlargement of the image formed at the focus of the telescope. The enlargement being 16 diameters, the image of the sun on this scale would have a diameter of one meter. The plate taken at the focus of the telescope has been enlarged to just the same size in order to facilitate a comparison between the two pictures.

The first set of photographs was made on July 17, 1906. *A* was made at 9^h 10^m and *B* with the enlarging camera half an hour later (Plate XV). *A* came out remarkably fine and clear, as appears from the plate, which shows distinctly the granules of the photosphere and many features of the spot. *B* did not come out so well as far as photographic art is concerned; but nevertheless it is far in advance of the other for representing the real features of the spot. There is no need of pointing out to the reader how much larger the nucleus is on *A*. As to the penumbra, the brightest part, which appears on *A* as forming the very brink of the inner edge, is clearly shown on *B* to shine at some distance from the nucleus; in the intervening space between the bright ring and the nucleus the sharp ends of the slender filaments are seen progressively fading away into the dark center.

The other set of photographs represents a sun-spot which was near the center of the sun on September 14, 1906. *A* was taken at 8^h in the usual way and enlarged to the size of *B*, viz., to 16 diameters. *B* was obtained as above, nearly half an hour later. As it is the best photograph I ever made of a sun-spot, and contains many interesting particulars, I have printed it in three graduated tones. The reader will certainly notice at first sight how very much larger the nucleus is on *A*. No one can fail to notice that the brightest part of the penumbra, which on *A* is bordering the umbra, is distinctly not so on *B*.

It being now clear that the smaller photographs, whatever their clearness may be, fail to show the minute features of a sun-spot, what is the cause of this deficiency? In my opinion it is to be found in the

PLATE XV



A. 17 th July 1906 B

A. 14 th September 1906 B

B. 14 th September 1906 B

1^{mm} = nearly $2''$

small size of the picture. The breadth of the filaments, especially at their sharper end, is, on the enlarged image, only two or three tenths of a millimeter. The same object on the image at the focus of the telescope must therefore have a size of only one or two hundredths of a millimeter. It does not seem that so narrow lines can be printed without confusion on a silver-bromide film. Consequently, where the filaments are shining more brightly their confusion causes a large patch of sensitive film to be blackened. At points where the sharpening of the filaments renders their light both narrower and fainter, their confusion prevents any action of the light to be marked on the film. The granules of the photosphere are very small, their diameter varying from 3″ to 1″ or a little under 1″, and yet they come out distinctly where the filaments of the penumbra appear only in patches. That is true; but the diameter of these granules, even those of only 1″, is still on our original smaller photographs three hundredths of a millimeter; that is, is about three times greater than the breadth of the filaments. Now, it is not very often, but only in the case of an exceptionally clear and calm atmosphere, that the minutest granules are clearly printed on the film. It is therefore not surprising if the filaments, which are considerably thinner, have as yet never been distinctly seen even on our best plates, where they form confused masses, somewhat like skeins of threads placed too far for distinct vision, as shown in Fig. *A*. But they do come out separately in Fig. *B*, since they are enlarged sixteen times *before* being impressed on the film.

On these photographs, series *B*, there are many particulars well worth being attentively considered. I have printed three copies on graduated tones, just to show as far as possible all the details visible on the plate itself. As it is much to be feared that a half-tone engraving, even of the best quality, as those in the *Astrophysical Journal*, will not reproduce the minute details, I have still enlarged the original plate threefold. On that last figure (Plate XVI) one second of arc is equal to 1.6 mm.

In many places around the penumbra the granules of the photosphere are evidently stretched forth in the direction of the filaments, especially on the south, at the origin of the bright "bridge" crossing the west part of the umbra.

At the outer edge of the penumbra there is a ring considerably darker than the middle or brighter ring, and even darker than the inner edge. But it is neither continuous nor identical all around. It is far more strongly marked in the north than in the south of the nucleus; and even in the north it is here and there cut with bright filaments.

Inside this ring comes the brighter part of the penumbra, the breadth and form of which are far from being uniform around the nucleus. Then comes the inner edge formed by the sharp ends of the filaments, which as they are narrowing begin at the same time to fade away from their previous brightness.

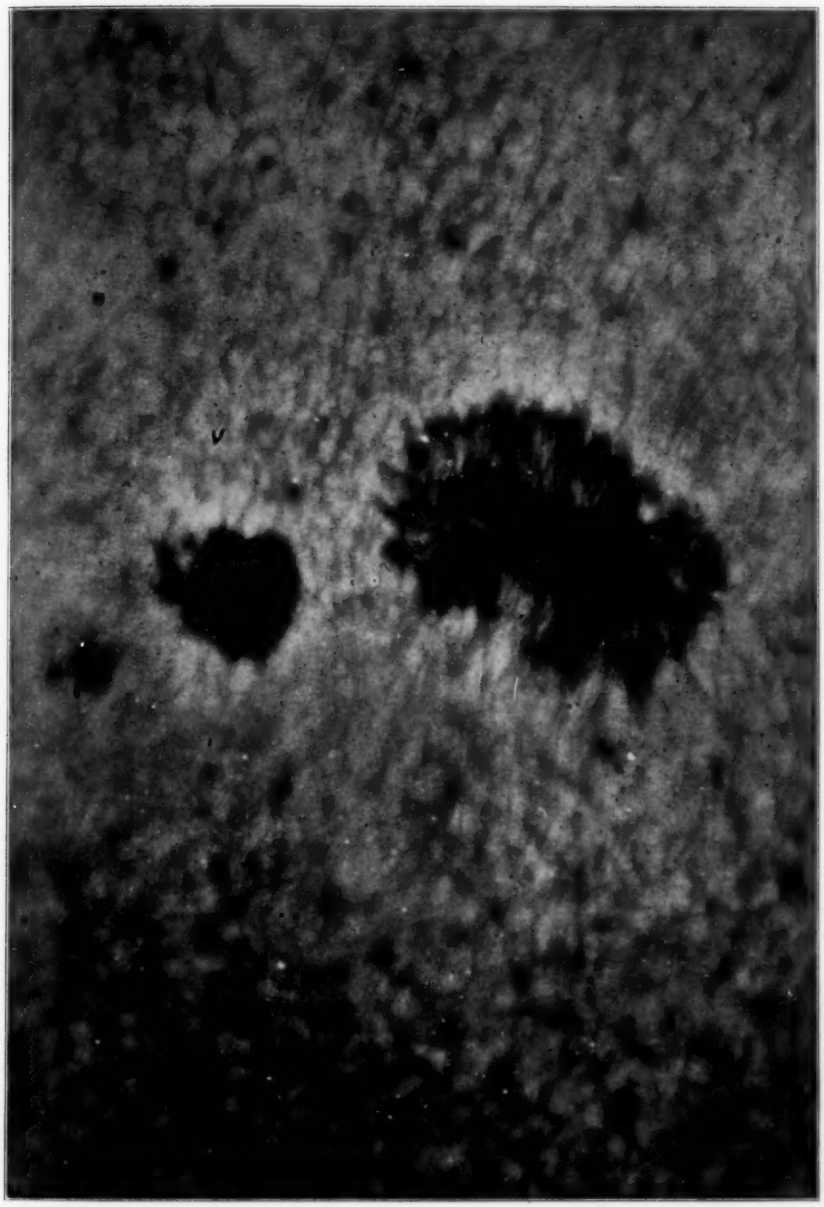
Let us suppose streamers of luminous matter, flowing from the outside toward the center, which would first bend somewhat downward as they cross the outer dark ring, then rise up again on the brighter ring, and lastly, on reaching the edge of the nucleus, begin to rush downward, apparently along a descending slope, and grow thinner and fainter as they gradually melt into it. As far as I can understand a drawing, these would show precisely the same form and bending which strike me when I consider the lines of the filaments.

This therefore seems to give strong support to the opinion that the dark nucleus is a hole in the photosphere; not, of course, a vacuum, but a hole filled up with materials which must be at the same time of lighter density and less brilliancy. The lighter density is shown by the fact that the filaments bend down into it, while on a fluid of greater density they would run upward.

I do not intend in any way to construct a theory of sun-spots, but only to point out what seems to be represented in these interesting photographs.

ZÔ-SÈ OBSERVATORY, NEAR SHANGHAI,
March 22, 1907

PLATE XVI



September 14, 1906. 1 mm = 0".6, nearly.

Minor Contributions and Notes

ON THE VELOCITY OF METALLIC PARTICLES IN THE SPARK DISCHARGE

The *Astrophysical Journal* for January 1907 contains a paper by Professor G. F. Hull in which certain experiments made by Mr. Hemsalech and myself[1] are referred to, and their explanation as given by us is called in question. Professor Hull's results have already been quoted as invalidating our conclusions; it seems therefore worth while pointing out that there is no real connection between the two sets of experiments. Our work led to the measurement of the velocities of metallic particles under certain conditions in a spark, and Professor Hull failed in his discharges to obtain a displacement of lines corresponding to these velocities; but our paper, as far as I know, contains no sentence which could have led anyone to expect a Doppler effect under the conditions under which Professor Hull experimented, and the fact that no displacement of lines was observed by him cannot therefore be taken to affect our results. If Professor Hull were right in his two contentions, (1) that metal particles of mercury and cadmium volatilized by the spark do not travel at a higher rate than 100 meters a second, and (2) that Mr. Hemsalech and I observed the propagation, not of matter, but only of a luminosity, he ought to have driven the argument to its logical conclusion, which is that a "luminosity" may carry the spectrum of a metal with it, while the metal particles themselves are left behind, giving up their spectral ghost to a phantom which rushes ahead. That his reasoning leads to this paradox I proceed to show.

Your readers may remember that Mr. Hemsalech and I worked with sparks sufficiently intense to produce singly a strong impression of the spectrum. Our method of operation indeed required this in order to secure the object of our research, which was indicated in the first two sentences of our paper in these words: "When an electric spark passes between metal electrodes, the spectrum of the metal

[1] *Phil. Trans.*, **193**, 189, 1899.

appears not only in immediate contact with the electrodes, but stretches often across from pole to pole. It follows that, during the short time of the duration of the spark, the metal vapors must be able to diffuse through measurable distances."

The problem is thus clearly stated. There is evidently no metal vapor in the spark-gap before the discharge has set in, and the first appearance of a metal line proves without question the presence of the metal which might possibly have traveled more quickly than the experiment indicates, and be rendered luminous only at a later stage; if it traveled more slowly, the paradoxical result already mentioned would follow. Our measurements therefore give the lowest possible values of the velocities, and give them with a directness which cannot be invalidated by the absence of any Doppler effects. In the case of cadmium vapor, to which Professor Hull more especially refers, the metal lines appear at a distance of 2 mm from the electrode about the millionth part of a second after the discharge has set in. This is certain because the position of the air lines fixes accurately the time of beginning of the spark. The "luminosity" has got to the center of the spark with an average velocity of about 500 meters per second, and it has found the metal there. The question therefore reduces itself to this. Can the luminosity show the lines of cadmium before the cadmium has reached the spot?

In referring to our own discussion of our results, it will be noted that we favored the view that the motion of the metal particles from the electrode to the center of the spark was a process of diffusion which follows the explosive volatilization of the metal by the spark; we actually showed by calculation that the order of magnitude was the same as that of the vapor treated as if it were flowing from a place of compression into a vacuum. It is clear, therefore, that we did not intend to apply our results to the case of continuous successions of discharges where ultimately some equilibrium in the distribution of the density of the metal vapor must be reached. But if we turn to Professor Hull's experiments, we find that, instead of using, as we did, single sparks taken from charged condensers, he worked with the discharges of the induction coil without capacity at all. Under these circumstances there was no reason to suppose that the effects noticed by us should be observed. Professor Hull does not

give sufficient information on the nature of the discharge he employed, but from the data given it is probable that it resembled that commonly observed in a vacuum tube. At any rate, I have frequently observed, under conditions which were not dissimilar, the cathode glow giving its characteristic spectrum in a sharp fine layer on the cathode, and the positive part of the discharge showing the bands of nitrogen. The electrodes would volatilize to some extent, but they would not be driven explosively into the center of the spark. The two sentences by which he attempts to bring his own experiments into connection with ours rest on an exceptionally bold piece of extrapolation for which there is no justification.

There are no doubt difficulties connected with the interpretation of some of our experiments, notably the function played by the metal in the oscillations of the discharge. Attention was drawn to certain appearances pointing to the fact that the metal vapor acted as conductor in the discharge, and that the oscillation of the spark produced periodic changes in the velocities of the metal particles. Mr. Royds is at present repeating our experiments with improved apparatus, and the preliminary results obtained show that his investigation will clear up some doubtful points. While I do not deny that when an electric discharge is established, the propagation of a "luminosity" may simulate to some extent the appearance of propagation of luminous matter, and that some observed effects may thus be accounted for, I think that a careful examination of the reproductions of photographs published in our paper might have convinced Professor Hull that such an explanation cannot be applied to our experiments. Is it likely, for instance, that a "luminosity" would set out from the pole with a large velocity which gradually diminishes and ultimately settles down to rest, unless it were connected with the transport of material particles?

ARTHUR SCHUSTER

VICTORIA PARK, MANCHESTER
March 24, 1907

NOVA T CORONAE OF 1866

This object, which suddenly blazed up to the 2d magnitude on May 12, 1866, and then faded to the 9½ magnitude, proved to be a star which had been previously observed by Argelander, being *B.D.*

+27°.2765. After this extraordinary outburst, it apparently returned to its original condition, at which it has ever since remained.

This is the first of the novae to be examined with the spectroscope. Observations were obtained of its spectrum when bright in 1866 by Huggins, who was just introducing the spectroscopic study of the stars.

As this is the only example of the novae which was known previous to its outburst—with the probable exception of *Nova Persei* of 1901 (See *H.C.O. Circular* No. 66), and as it is the brightest of these bodies at present visible, it is important that we should study it carefully, in the hope that some information as to the cause of these outbursts may be gained. The present observations are therefore intended to be a small contribution to the history of the star at the present time.

Some of the other novae which are still visible present a hazy or out-of-focus image in a large telescope, viz., *Nova Cygni* of 1876, *Nova Aurigae* of 1892, *Nova Sagittarii* of 1898, etc.

To see if anything peculiar existed in the telescopic image of *T Coronae*, I have examined it several times of late years with the 40-inch telescope. Its focus was measured with respect to that of the 9^{m}.1 star *B.D.*+26°.2761, which precedes it 1 minute of time, and is 1½′ north. The star was also examined inside and outside the focus. There was nothing in these observations in any way to suggest a difference from the ordinary star. We must, however, remember that, so far as the extra focal images of the novae are concerned, the peculiarity has not usually made its appearance until the star has become much fainter than the present brightness of *Nova Coronae*. The strong visual light may therefore mask this peculiarity in the case of this star.

The following focal measures were carefully made:

		Inches
1903 May 4.	Focus on *Nova*. Scale	= 2.22 (3)
	on *B.D.*+26°.2761	2.21 (3)
	Diff.	+0.01 inch

Of course, this quantity is too small to be a real difference in a focal determination with the 40-inch telescope, though it is in the right direction—being a little farther from the object-glass—for a nova.

The image was also tested on several other dates, though no measures were made. The star was also observed for color:

1904 May 4. "There is no color to the star."

1906 Aug. 28. "It is almost colorless, but the seeing is too bad to be certain." Later: "So far as I can judge, the star is white."

1906 Sept. 11. "It may be tinged with yellow, but not certain. It is not *white*."

For the brightness of the *Nova* the following observations were made:

1906 Aug. 28. Independent estimate $= 9^{m}3$.
Aug. 28. $\frac{2}{10}^{m}$ less than $+26°2761$
Sept. 11. $\frac{3}{10}^{m}$ less than $+26.2761$

In the *B.D.* these two stars are:

$+26°2761 \quad 9^{m}1$
$+26.2765 \quad 9.5\ (T)$

It would seem, therefore, that the *Nova* is now essentially of the same brightness it was before the outburst in 1866.

These two stars are Cambridge *A.G.C.*, 7412 and 7433, and their places, brought up to 1906.0, are:

	$15^{h}\ 54^{m}\ 32^{s}.39$	$+26°\ 12'\ 38''.8$
T	15 55 34.32	+26 11 10.2
	$\Delta\alpha + 1^{m}\ 1^{s}.93$	$\Delta\delta - 1'\ 28''.6$

On 1906 August 28, I compared these two stars for position of the *Nova:*

$\Delta\alpha = +1^{m}\ 1^{s}.85$ (8 transits)
$\Delta\delta = -1'\ 27''.1$ (2 measures)

It will be seen that my measures make the $\Delta\delta$ $1''.5$ smaller than the value given by the Cambridge observations. If, however, this error is divided between the two stars in the Cambridge observations, the discordance would not be greater than would be expected from good meridian positions. So there seems to be no indication of motion in the *Nova*.

A NEBULA NEAR THE *NOVA*

While examining the star, I found a faint nebula in the field with it, following. The nebula is of the 14th or 15th magnitude, and is from 5″ to 10″ in diameter, without any nucleus.

The following measures of this nebula and the *Nova* were made:

		$\Delta\alpha$		$\Delta\delta$
1903	April 27	$+4'\ 50\overset{s}{.}9$ (1)	$=+0^m\ 21\overset{s}{.}62$	$+1'\ 13\overset{''}{.}1$ (1)
	April 28	$+4\ 50.5$ (1)	$=+0\ 21.60$	$+1\ 13.4$ (1)
1906	Sept. 11	$+4\ 49.1$ (2)	$=+0\ 21.48$	$+1\ 11.1$ (2)

The discordances in these results are purely accidental, from the faintness and indefiniteness of the nebula.

From these observations, and the Cambridge position of the *Nova*, the place of the nebula is:

$$1906.0 \quad \alpha = 15^h\ 55^m\ 55\overset{s}{.}85; \qquad \delta = +26^\circ\ 12'\ 21\overset{''}{.}9.$$

E. E. Barnard

Yerkes Observatory
April, 1907

REGULAR AND DIFFUSE REFLECTION

In the *Astrophysical Journal*, **24**, 351, 1906, the writer is quoted as having found that "various common minerals which reflect only *diffusively* in the region of the spectrum of shorter wave-lengths than 8 μ have bands of metallic reflection from 8.5 to 10 μ." Since then the original paper is being interpreted from the standpoint of "diffuse" and "metallic" reflection. The whole is based upon a misconception and misquotation of the original statement,[1] viz.: "The writer found the reflecting power [of various silicates] to be practically zero for the region of the spectrum up to 8 μ, followed by bands of metallic reflection from 8.5 to 10 μ."

Since the surfaces examined were in nearly all cases plane and highly polished, there was no "diffuse" reflection, which is an entirely different question from low, "practically zero," reflection; and it would indeed be very unfortunate to have such an idea (that diffuse and low reflection are synonymous) gain foothold. It is with the hope that such hazy notions may be cleared up that the present comments are written.

When energy is reflected from a plane surface, it is commonly called "regular" (or, less accurately, "specular") reflection. On the other hand, energy reflected from a rough surface suffers "diffuse" reflection. In "diffuse reflection" for each infinitesimal surface the

[1] *Phys. Rev.*, **23** 247, 1906.

ordinary laws of reflection are obeyed in full, unless the linear dimensions of the reflecting surface, or of the rugosities or inequalities on it, are small compared with the wave-length. However, the unpolished surface as a whole destroys all phase-relation between the particles in the reflected wave-front, which is no longer plane, but irregular.[1] This irregularity decreases as the angle of incidence increases, so that for a given roughness we get regular reflection. The long waves will be reflected first, then the shorter. "Smoked glass, which at perpendicular incidence will show no image of a lamp at all, will at nearly grazing incidence give an image of surprising distinctness, which is at first reddish, becoming white as the angle increases."[2]

The amount of energy reflected "regularly" from a plane surface will depend upon the reflecting power of the substance. Now, the reflecting power, R, of any substance is related to its index of refraction, n, and its absorption coefficient, k, by the equation

$$R=\frac{(n-1)^2+k^2}{(n+1)^2+k^2}.$$

For "transparent media," i. e., "electrical non-conductors,"[3] the absorption coefficient is so low that it is negligible, and the reflecting power is a function of only the refractive index. Here the reflecting power is low, only 4 to 6 per cent., and decreases with increase in wave-length. All transparent media thus far examined (except silver-chloride) show bands of selective reflection. In these bands the absorption coefficient, k, attains high values.

If k becomes sufficiently large,[4] of the order unity, the absorption affects the reflecting power, and the heat- and light-waves no longer enter the substance, but are almost totally reflected, as in metals; whence the name "bands of metallic reflection." For metals, "electrical conductors," he absorption coefficient is so large that nearly all the energy, for nearly all wave-lengths, is reflected.

If, then, the eye were sensitive to heat-waves, many substances would have a "surface color" similar to that of gold and fuchsine

[1] See Wood's *Optics*, p. 36.

[2] *Ibid.*, p. 37.

[3] See Drude, *Optics;* also Schuster, *Optics;* Pockels, *Lehrbuch der Crystallographie.*

[4] See Schuster, *Optics*, p. 259; Pockels, *Crystallographie*, 1906.

in the visible spectrum. In other words, the reflecting power of plane surfaces ("regular reflection") of "transparent media" will be low for all regions except where there are bands of "metallic reflection." It is evident that the diffuse reflection from rough surfaces of transparent media will also be selectively reflecting. Hence in the stream of energy reflected in any direction the density will be greatest for wave-lengths 8.5 to 10 μ, for silicates. In this case, however, the term "metallic reflection" is not to be used synonymously with the expression "specularly reflected," as it appears to be in this *Journal* (*loc. cit.*, pp. 351 and 353); for the latter refers to "regular reflection." Since the energy-density in every direction must be greatest for wave-lengths 8.5 to 10 μ, one would expect to detect this difference in any direction, and not simply at the angle of regular reflection.

As a whole, then, what the writer found is that the silicates reflect like transparent media for wave-lengths up to 8 μ, and like metals from 8.5 to 10 μ. In other words, it may be said that in quartz, SiO_2, the silicon ions retain the proper period of undamped electrical vibration which they would have in the element silicon (which has metallic as well as non-metallic properties), for wave-lengths 8.5 and 9.03 μ, while for all other wave-lengths the vibration periods are more damped and the reflecting power is decreased.

W. W. COBLENTZ

BUREAU OF STANDARDS, WASHINGTON
March 26, 1907.

VARIABLE RADIAL VELOCITY OF *U CEPHEI*

Measures of plates of this star, made with the single-prism spectrograph, show a variation of 95 kilometers in the radial velocity. This star is classed with *Algol* variables, and its light-variation is recognized as somewhat peculiar. It has a period of 2.5 days and a variation from the 7.1 to the 9.2 magnitude. The spectrum is of the early hydrogen type. There is indication of complexity in the lines, suggesting that both components of the system may be bright. More plates will be needed to give this interesting question a definite answer.

V. M. SLIPHER

THE SPECTRUM OF ϵ *CAPRICORNI*

Spectrograms of this star made last autumn, with the single-prism spectrograph, show its spectrum to contain bright lines. At Harvard the star was found to have a peculiar spectrum, but the bright lines were not recorded. My previous plates of the star, from which its variable velocity was discovered, were made with high dispersion and were, owing to this and their narrowness, unsuited to reveal the bright lines. On the late plates the hydrogen lines are paired—the sharp, dense dark line with a weak bright line above it. As is usual with this type of peculiar spectrum, the emission line is more intense in the lower members of the hydrogen series. On the recent plates, which unfortunately extend over only a short period of time, the dark hydrogen and metallic lines were displaced toward the red by an amount corresponding to a velocity of 45 kilometers on October 8, which had decreased to 35 kilometers by October 27. In addition to these dark lines the spectrum contains also helium absorption lines, which are, however, very broad and indefinite, and do not share with the other dark lines the shift toward the red. Measures on some of the less diffuse ones indicate, on the other hand, a small displacement toward the violet. It seems probable, therefore, that the dark hydrogen and metallic lines belong to one component, and the diffuse helium and bright hydrogen lines are due to the other component, and that the weak bright lines are blotted out by the intense dark ones, except when the relative velocity of the two stars is sufficient to separate well the two lines.

V. M. SLIPHER

LOWELL OBSERVATORY, FLAGSTAFF, ARIZ.
February, 1907.

Reviews

Generalkatalog der Potsdamer photometrischen Durchmusterung des nördlichen Himmels, Enthaltend die Gröszen und Farben aller Sterne der *B. D.* bis zur Grösze 7. 5. Von G. Müller und P. Kempf. *Publicationen des Astrophysikalischen Observatoriums zu Potsdam*, Nr. 52, Siebzehnter Band, 1907.

Following closely Part IV, containing the final zone measures, comes the general catalogue giving the revised magnitude and color results for the northern heavens, with the stars arranged in order of right ascension. This noble volume contains 14,199 stars, the various columns giving data as follows:

1. Current number.
2. *B. D.* number.
3. R. A. for 1900 to seconds of time.
4. Declination for 1900 to tenths of a minute of arc.
5. *B. D.* magnitude.
6. Measured magnitudes, usually two, one each by Müller and Kempf.
7. Photometer used.
8. Mean estimated color.
9. Mean magnitude.
10. Remarks, giving Bayer's letter or Flamsteed's number; Struve's or J. Herschel's number for doubles.

The exceeding usefulness of this general catalogue puts the astronomical world under great obligations to its authors; but the valuable introduction is equally noteworthy and deserves a more extended notice than can be given here. The catalogue is not a mere rearrangement of the results of the different zone catalogues; but these have been discussed, their mutual relations investigated, and systematic corrections applied so as to make from the four parts one homogeneous whole. All this has been carried out with the scientific thoroughness and critical ability which are characteristic of the staff of the Potsdam Observatory. The completed work therefore stands alone in the literature of stellar photometry, and does much toward putting that branch of the science on a level with the astronomy of position.

The discussion in the introduction falls under two heads: first, the corrections to be applied to the measures by the two observers and with the four different arrangements of the photometers, with respect to

magnitude and color; second, a comparison of the magnitudes, thus corrected, with the various Harvard catalogues. The original observations were planned to furnish the data required for the comparisons first mentioned. An equal number of observations were made on each star by each observer, and the mean used, so that only the difference in instruments needed consideration. Two photometers were used: the smaller, D, with an objective of 135 mm aperture; the larger, C, with objectives of (I) 67, (II) 36.5, and (III) 21.5 mm. Since the magnitudes of the 144 fundamental stars had been determined with D, the results obtained with C were reduced to that standard. The systematic differences of magnitude between the two photometers, due to star colors, were so small, 0.01 and 0.02, as to be negligible. On the contrary, the progression due to magnitude was pronounced; the corrections to C_I ranging from −0.09 at magnitude 3.5 to +0.09 at magnitude 7.4, and the corrections to D ranging from −0.11 at 4.8 to +0.11 at 9.0. These differences seemed due to the relative brightness of the standard and measured stars in each instrument; the standards used with C averaging 5.4, those used with D 6.8. The corrections applied to C_{II} and C_{III} showed no progression, and were 0 and −0.03 respectively.

The color estimates were also reduced to the system of photometer D, the corrections applied to C_I and C_{II}, in terms of the small Potsdam steps (18 between white and red), increasing from 0 for white stars to 2 for the more intensely colored ones. A comparison with Osthoff's color-scale (799 stars in common) showed the Potsdam step equivalent on the average to 0.4 of Osthoff's unit, but this quantity varied from 0.1 to 1.1 in the means for the different groups.

Before comparing with Pickering's results, it was necessary first to investigate the relations between the various Harvard catalogues. Although Vol. **45**, like the Potsdam Durchmusterung, contains all the stars to magnitude 7.5, it was found impossible to compare with it directly, as the magnitudes of 5217 of its stars were taken from older catalogues, and considerable systematic differences were found between the various volumes, thus depriving Vol. **45** of the character of a homogeneous *Durchmusterung*. After deducing these systematic differences, ranging from 0 to 0.2 or 0.3, the brighter Potsdam stars, measured with C_I and C_{II}, were compared with Harvard **44**, and the fainter stars measured with D, with Harvard **45**. The following formula expresses the difference between the two systems:

$$\text{Potsdam-Pickering} = +0^m.229 + 0^m.024\,m - 2^m.25 - 0^m.027C - 0^m.008\,m - 2^m.25\,C,$$

in which m indicates magnitude and C color (white, 0; yellowish white, 1; whitish yellow, 4; and yellow, 6). The residuals left after applying this

formula are tabulated on page xxxii. They range from 0 to 0.09 magnitude, the mean value, without regard to sign, being 0.02 for groups containing on the average 113 stars.

The general conclusions may be condensed as follows:

1. Accidental errors in measures with the Zöllner photometer can be confined to a few hundredths of a magnitude.

2. Systematic errors, in spite of the utmost precautions, may amount to $0^m.1$ per magnitude.

3. Each series of photometric measures should be corrected for systematic differences before being combined with another series.

4. Star colors have a greater effect on magnitude measures than had been supposed.

5. Visual estimates of star colors apply only to the particular eye and instrument used.

Though not mentioned by the authors, it occurs to the reviewer that the above conclusions suggest that further advances in stellar photometry, especially in the allowance which must be made for color, can best be made by photographic methods.

J. A. PARKHURST

Cours d'Astronomie. Première partie: Astronomie théorique. Par H. ANDOYER. Paris: A. Hermann, 1906. Pp. 221, with numerous diagrams. Fr. 9.

This first part of Professor Andoyer's work is divided into fifteen chapters. It is not printed from type, but reproduces a very distinctly hand-printed manuscript. The principal object of the book is stated by the author to be the study of the apparent motions of the celestial bodies. After an introductory chapter on spherical trigonometry, the author considers the various systems of co-ordinates used in astronomy (chapters 2–5). To refraction, parallax, and aberration the succeeding three chapters are devoted. Chapter 9 gives a short initiation into the principles of celestial mechanics, upon which he draws not inconsiderably in chapters 10–12, which treat respectively of the theory of precession and nutation, and the apparent motion of the sun, with special reference to the notion of time. The last three chapters (13–15), to which group in a fashion parts of chapter 12 likewise belong, deal with the motion of the movable celestial bodies, sun, moon, planets, and satellites, and the last chapter in particular with the eclipses of these bodies.

A book which comes from the pen of so eminent a scholar as Professor Andoyer has of course in a high degree those remarkable qualities for

which the French textbooks have become the exponents *par excellence:* clearness and elegance. If in what follows attention is called to some minor shortcomings, they are offered with proper appreciation of those excellent qualities. The book is evidently intended for beginners; otherwise one could not understand the entire absence of references from cover to cover. The knowledge of calculus seems to be all that the author presupposes in the way of prerequisites. There is but one paragraph where higher attainments on the part of the reader are called into service—in the ninth chapter, where the author speaks of the convergence of the Fourier series and arrives at Laplace's and Rouché's result. This might well have been left out, since it is decidedly beyond the understanding of the average reader. Instead, a more detailed use of the fundamental principles of analytical mechanics might have added much to the understanding of this chapter. The author probably deliberated a good deal where to place the contents of this chapter to best advantage. Brünnow succeeded well in giving the matter a place in the early part of his book. The present arrangement likewise has advantages, and, after all, such matters depend a good deal upon the personal predilections of the author.

The very complete introduction to spherical trigonometry, which comprises many details for purely geodetic students, creates the impression in the reader that this line of investigation will be taken up in the further issues of Andoyer's work. But the next following chapter somewhat disillusions him, when the short paragraph given to the ellipsoidal figure of the earth is considered. The custom which prevails in some textbooks, of considering "reduced" latitude and "geocentric" latitude as synonymous terms, is adopted in this treatise too. This is unfortunate, especially since Helmert's fundamental treatise distinctly defines reduced latitude in precisely the manner in which the eccentric anomaly is defined in the elliptic motion. Andoyer's "auxiliary" angle is Helmert's reduced latitude. A figure which shows distinctly the three different kinds of latitude would be quite in place. The figure given in the text does not convey clear conceptions, since a point M at an elevation MM' is made the starting-point for the discussion. It is the surface point to which the figure should refer.

In conclusion, attention is called to the contents of the last three chapters, which in previous textbooks on the same subject have received but scant recognition, and which open the way to a more vivid conception of the subject-matter that evidently will follow this very excellent first part of Professor Andoyer's book.

KURT LAVES

MARCH 19, 1907

Physical Optics. By ROBERT W. WOOD. New York: The Macmillan Company, 1905. Pp. xiii+546; figs. 325 and four plates. $3.50.

Every reader of Professor Wood's *Physical Optics* must be impressed with the value of the book as a compendium of the best modern views on optical phenomena. And it is a great satisfaction to find a book so full of the most valuable theoretical and experimental data which is written in a clear, forceful, and original style, always from the standpoint of the physicist rather than from that of the mathematician or the mere statistician.

Although the author makes no claim to originality in the mathematical treatment, he is to be congratulated on his excellent choice and arrangement of the mathematical material, and on the clear, concise, and illuminating discussion with which he has amplified it. The general methods of Verdet are followed in the earlier chapters, while specific paragraphs are credited to their various originators: e.g., Airy's treatment of diffraction by a circular aperture, Runge's theory of the concave grating, and Michelson's theory of the echelon. Drude's unsurpassed treatment, based on the electro-magnetic theory, has been closely followed in some of the later chapters.

One of the most interesting chapters in the book is that on "The Theory of Dispersion." The Cauchy, Sellmeier, Helmholtz, and Ketteler-Helmholtz formulae are discussed in order, and followed by Drude's treatment from the electro-magnetic standpoint. The fascinating experimental results of Rubens, Nichols, Aschkinass, Pflüger, and of Professor Wood himself, are given in illustration and verification of the formulae.

The chapters on "Optical Properties of Metals," "Rotatory Polarization," and "Magneto-Optics" are rich in valuable theoretical and experimental material, and the chapter on "Transformation of Absorbed Radiation" includes the remarkable results recently obtained by Professor Wood with sodium vapor. The chapter on "The Laws of Radiation" is also one of great interest, including a brief statement of the theoretical results of Larmor, Stefan, Boltzmann, Wien, and Planck, and the experimental results of Nichols and Hull, Lummer, Pringsheim, Rubens, Kurlbaum, Pflüger, and others.

One cannot read the book without being impressed anew with the large amount of wonderfully ingenious qualitative work which Professor Wood has done, especially in the domain of anomalous dispersion and the optical properties of metallic vapors.

The author apologizes in his preface for the large number of references to his own work, but one is glad that modesty did not prevent him from

including them, as the collecting in this way of so much of his work is one of the valuable features of the book. The several references to the "Schlieren-Methode" of Töpler, and the author's photographs of sound-waves by this method, although illustrating the points in question very well, would, however, perhaps be more in place in a general textbook than in a work on physical optics. The description of a lecture-table experiment for illustrating the phenomena of mirage and a two-page account of a mica echelon which, as he says, is "of no use as a piece of optical apparatus" and "useless as a tool for research," seem to be somewhat below the high scientific plane of the book as a whole. There are the usual number of typographical errors which creep into a first edition. They are especially conspicuous in the formulae of the chapter on "The Theory of Dispersion."

The thoroughly up-to-date character of the book is evidenced by the chapters on "The Laws of Radiation," "Transformation of Absorbed Radiation," "The Nature of White Light," and "The Relative Motion of Ether and Matter." Numerous references are found throughout the book to articles which appeared as late as 1905.

The book should be read to be properly appreciated, and it should be read by every student who wishes to be thoroughly informed on what is best and latest in physical optics. It will perhaps prove to be one of the most valuable of the many contributions for which the science of physics is already indebted to Professor Wood.

H. G. G.

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